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EFFECTS OF CHARGE DENSITY RISE TIME
UPON CERENKOV RADIATION

by

Perry M. Suttle

June 1990

Thesis Advisor J.R. Neighbours

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This study investigates the effects of different charge density rise times upon the magnitude of the Cerenkov radiation produced by a semi-infinite electron beam. The magnetic field pulses were generated for different rise times and relationships between the maximum magnitudes and their associated rise times were obtained. These were compared with theoretical relationships derived from a power series approximation. The generated results were close to those predicted by theory for short rise times at short radial distances. For longer rise times, the departure from theory was caused by the magnitude of the decay portion of the magnetic pulse. This effect could be mitigated at longer rise times by increasing the radial distance from the electron beam.

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Effects of Charge Density Rise Time Upon Cerenkov Radiation

by

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ABSTRACT

This study investigates the effects of different charge density rise times upon the magnitude of the Cerenkov radiation produced by a semi-infinite electron beam. The magnetic field pulses were generated for different rise times and relationships between the maximum magnitudes and their associated rise times were obtained. These were compared with theoretical relationships derived from a power series approximation. The generated results were close to those predicted by theory for short rise times at short radial distances. For longer rise times, the departure from theory was caused by the magnitude of the decay portion of the magnetic pulse. This effect could be mitigated at longer rise times by increasing the radial distance from the electron beam.

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I. INTRODUCTION

A. BACKGROUND

Cerenkov radiation is the electromagnetic emission caused by the passage through a dielectric medium of a charged particle that is traveling faster than the speed of light in that dielectric medium.

Imagine an electron moving through a dielectric medium relatively slowly. The electric field of the moving electron tends to repulse the electrons of nearby atoms while attracting the nuclei, creating distortions in the atoms which results in the creation of temporary dipoles. Thus, the medium around the electron becomes polarized. Because the polarization field around the electron is completely symmetric, there is no resultant field at large distances. [Ref. 1: pp. 3-4]

But if the electron is moving faster than the speed of light in the dielectric medium, the polarization field is no longer completely symmetrical. While the azimuthal plane symmetry is preserved, there is a resultant dipole field along the axis of travel which is apparent at large distances. This field is created at each path element along the electron's track. Each field radiates a brief electromagnetic pulse before the atoms realign themselves and return to their normal shape. [Ref. 1: p. 4] This behavior can be seen in Figure 1.

The time development of Cerenkov radiation was introduced by Buskirk and Neighbours for application to electron beams. Single charged particles produce a radiated power that is proportional to the frequency. Electrons in an accelerator bunch radiate coherently, an effect that more than offsets a single particle increase in radiated power with frequency. [Ref. 2: p. 3750]

This time structure effect is not observable with present technology in *S* or *L* band linear accelerators due to their relatively high fundamental frequency. Induction accelerators should produce observable results in air for energies greater than 25 MeV due to their longer electron bunch structure. [Ref. 2: p. 3753]

Lyman conducted a preliminary study investigating the magnetic field radiated from a passing charge bunch traveling over a finite path. She showed that the magnetic field magnitude versus time plots depended not only on the observer's position but on the specific time conditions of each case. [Ref. 3: p. 3]

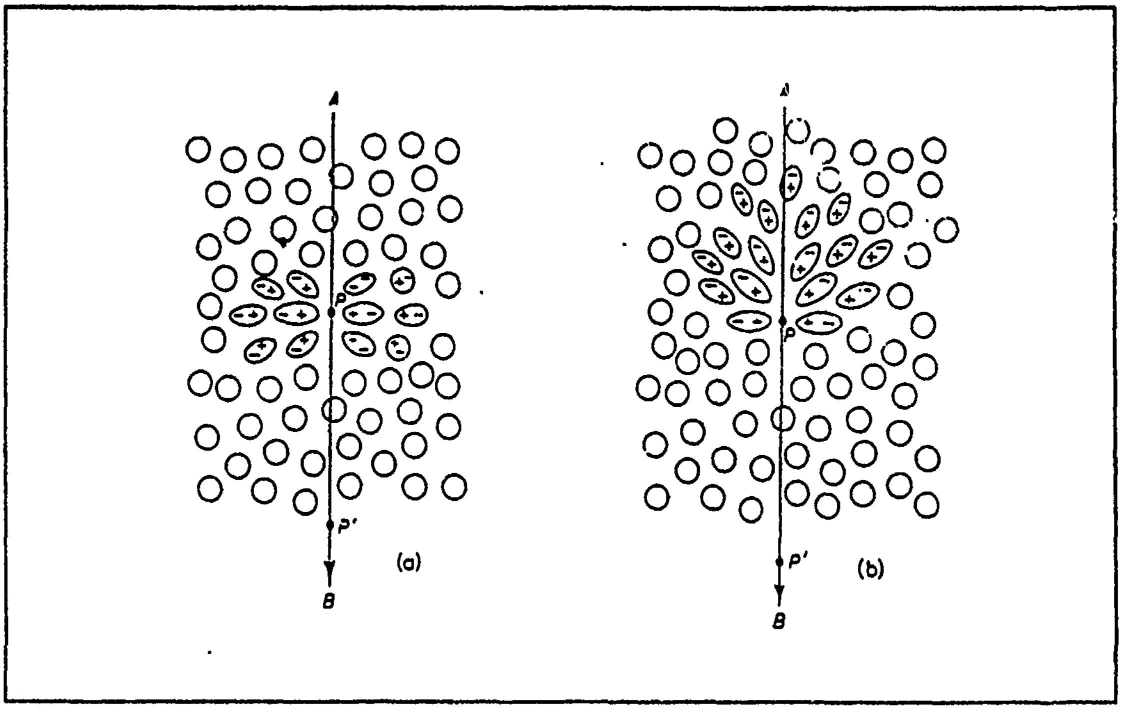

Figure 1. A Charged Particle Moving Through a Dielectric Medium: (a) slow speed (b) fast speed [Ref. 1: p. 4]

Price expanded on Lyman's study by investigating how the magnitude and shape of the radiated field depended on the observer location relative to the time boundaries defined by relationships between the arrival times of different parts of a pulse. [Ref. 4: p. 3]

B. OBJECTIVES

One of the key assumptions in the Cerenkov radiation time development is that the current pulse has a ramp-front, i.e. that the electron pulse density increases from zero to a maximum constant value in a linear fashion. The end of the pulse is assumed to be a mirror image of the ramp-front but the effects of the pulse end have not been considered in these studies.

The primary objective of this work is to investigate the effects of varying the rise time of the electron density, i.e. varying the slope of the ramp-front, upon the magnitude of the Cerenkov pulse.

Secondary considerations are to compare the shape and magnitude of Cerenkov pulses along lines parallel to the axis of electron beam travel, evaluate the maximum

Cerenkov magnitude as a function of radial distance from the axis of electron beam travel, and plot the magnetic field as a function of time and radial distance.

II. THEORY

A. RADIATION FIELDS

To calculate the Cerenkov magnetic and electric fields, one must first determine the potentials from the moving charge distribution. The relationship between the fields and potentials are given by Equations 2.1 and 2.2 [Ref. 2: p. 3750], which are

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{2.1}$$

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c_o}\frac{\partial \mathbf{A}}{\partial t}. \tag{2.2}$$

The speed of light in free space is c_o while the speed of light in the medium is c. The electron pulse is assumed to move with velocity v in the positive z direction. The charge density ρ_v is assumed to be concentrated along the z axis while the charge is assumed to move with no change in shape [Ref. 2: p.3750] such that

$$\rho_v(\mathbf{r},t) = \rho(z,t)\delta(x)\delta(y), \tag{2.3}$$

$$\rho(z,t) = \rho_o(z - vt). \tag{2.4}$$

The charge density ρ_v has units of charge per volume while ρ and ρ_o have units of charge per lenght.

Using the assumption of Equation 2.3 and applying it to the usual retarded solutions of the wave equations, one obtains the potentials [Ref. 2: p. 3750]

$$\Phi(\mathbf{r},t) = \frac{1}{\varepsilon}\int \frac{1}{R}\,\rho(\mathbf{r}',t')dz', \tag{2.5}$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}}{c_o}\int \frac{1}{R}\,\rho(\mathbf{r}',t')dz'. \tag{2.6}$$

Note that ε is the permittivity of the medium. Also, t' is the retarded time and $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ such that

$$t' = t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}. \qquad (2.7)$$

The assumption of Equation 2.4 can now be inserted into the potential and a new variable $u(z') = z' - vt'$ can be introduced [Ref. 2: p. 3750] so that

$$\Phi(\mathbf{r},t) = \frac{1}{\varepsilon} \int \frac{1}{R} \rho_o(u) dz', \qquad (2.8)$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}}{c_o} \int \frac{1}{R} \rho_o(u) dz'. \qquad (2.9)$$

The new variable $u(z')$ can be written out explicitly where x, y, and z represent the coordinates of an observer or observation point and z' is the coordinate of the front edge of the electron pulse such that

$$u(z') = z' - vt + \frac{v\{x^2 + y^2 + (z - z')^2\}^{\frac{1}{2}}}{c}. \qquad (2.10)$$

The velocity $\mathbf{v}$ is in the positive z direction and $\mathbf{A}$ is proportional to $\mathbf{v}$. Therfore since $\mathbf{A}$ has a component only in the z direction, $\mathbf{B}$ only has components in the x and y directions and can be calculated from Equation 2.1 [Ref. 2: p. 3751] such that

$$B_x = \frac{v}{c_o} \int \frac{\partial}{\partial y} \left[\frac{\rho_o(u)}{R} \right] dz' + \frac{v}{c_o} \int \frac{1}{R} \frac{\partial}{\partial y} [\rho_o(u)] dz', \qquad (2.11)$$

$$B_y = -\frac{v}{c_o} \int \frac{\partial}{\partial x} \left[\frac{\rho_o(u)}{R} \right] dz' - \frac{v}{c_o} \int \frac{1}{R} \frac{\partial}{\partial x} [\rho_o(u)] dz'. \qquad (2.12)$$

The first integrals in Equations 2.11 and 2.12 have a falloff of R^{-2} at large distances for radiation and will be ignored. Implementing the u dependence on x and y (Equation 2.10 where $\partial u/\partial y = y/R$ and $\partial u/\partial x = x/R$) into these equations gives Equations 2.13 and 2.14 [Ref. 2: p. 3751] in the form

$$B_x = \frac{v^2}{cc_o} \int \frac{y}{R^2} \rho_o'(u) dz', \tag{2.13}$$

$$B_y = -\frac{v^2}{cc_o} \int \frac{x}{R^2} \rho_o'(u) dz'. \tag{2.14}$$

Note that the new term $\rho_o'(u)$ in Equations 2.13 and 2.14 is the derivative of $\rho_o(u)$ with respect to the function u, which was defined earlier. Equations 2.14 and 2.15 can be combined into one equation. Also a new function s can be defined as $s = (x^2 + y^2)^{\frac{1}{2}}$ [Ref. 2: p. 3751] such that

$$B = \frac{v^2}{cc_o} \int \frac{s}{R^2} \rho_o'(u) dz'. \tag{2.15}$$

B. TIME DEVELOPMENT

To evaluate Equation 2.15, one must consider the dependence of u upon z' as given in Equation 2.10. If Equation 2.10 is plotted in the u-z' plane, the first two terms are a straight line with unit slope and an intercept that changes with time. The third term is a hyperbola which opens in the $+\ u$ direction with asymptotic slopes of $\pm v/c$. In the Cerenkov case where $v > c$, the curve is as shown in Figure 2. The curve translates downward to smaller values of u as time increases. This is due to the time term in Equation 2.10. [Ref. 2: p. 3751]

Certain assumptions about the charge shape and configuration must now be taken into account. The charge is assumed to move with no change in shape. This was reflected in Equation 2.4: $\rho(z,t) = \rho_o(z - vt)$. Since ρ and ρ_o are also functions of u, $\rho(u) = \rho_o(u)$. The charge density is assumed to have a ramp-front profile, i.e. the density increases from zero to a maximum constant value in a linear fashion. The time that it takes for the charge density to reach this constant plateau is called the rise time [Ref. 4: p. 13].

Only changing currents (those with a nonzero $\rho_{o'}$) will contribute to the magnetic fields of Equation 2.15. If the charge density has a linear rise to a constant value, then the derivative $\rho_o'(u)$ will be a constant valued square pulse of magnitude ρ_m' as shown in Figure 3. [Ref. 2: p. 3751]

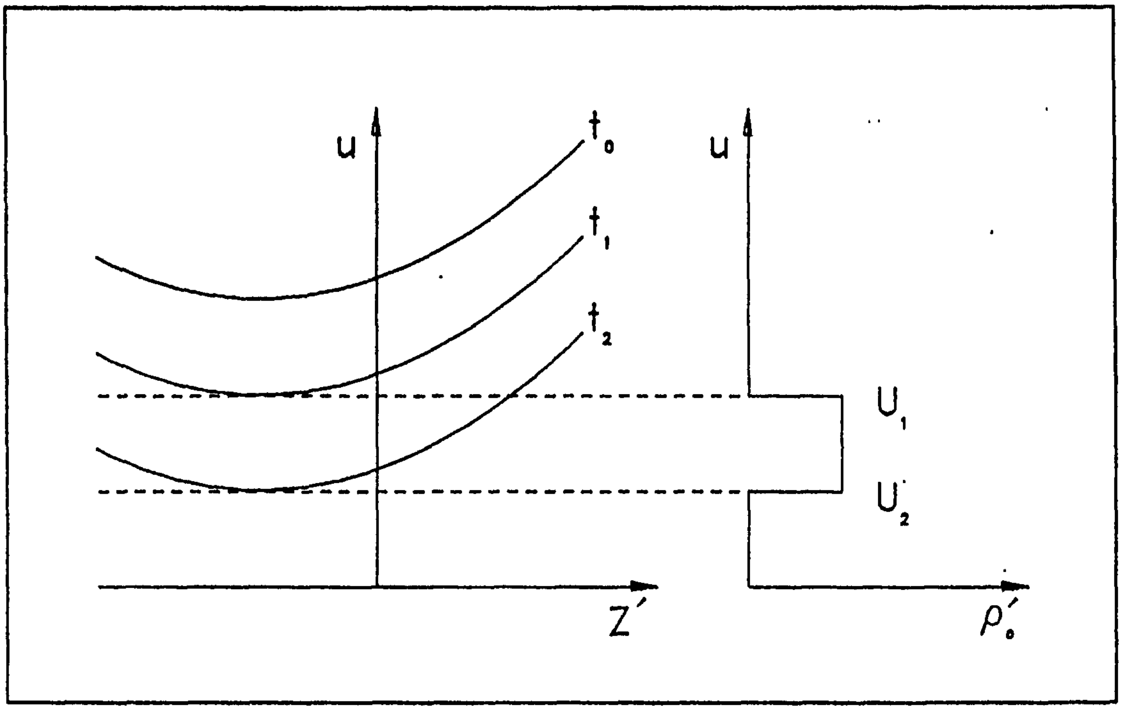

Figure 2. The Function $u(z')$

At large negative times, the $u(z')$ curve t_o (Equation 2.10) is completely above the nonzero portion of the $\rho_o'(u)$ curve so that the contribution from $\rho_o'(u)$ is zero and thus the magnetic field is zero. The $u(z')$ curve moves downward as time increases until the B pulse begins when $u(z')$ is tangent (curve t_1) to the upper portion of the of the $\rho_o'(u)$ pulse. The magnitude of the magnetic field increases as $u(z')$ moves downward with increasing time until the $u(z')$ curve becomes tangent (curve t_2) with the lower edge of the $\rho_o'(u)$ pulse. The nonzero part of Equation 2.15 has its largest extent at this time--from $u(z_2)$ to $u(z_3)$. At later times, Equation 2.15 breaks into two regions of the z' axis and the B pulse decreases with increasing time because the extent of the integral in the two regions decreases as a result of the upward turn of the $u(z')$ curve. [Ref. 2: p. 3751]

If the derivative $\rho_o'(u) = \rho_m' =$ constant, then it can be taken outside the integral of Equation 2.15. Also the values of $n = c_o/c$ and $\beta = v/c_o$ can be inserted so that

$$B = n\beta^2 \rho_m' \int \frac{s}{R^2}\, dz'. \tag{2.16}$$

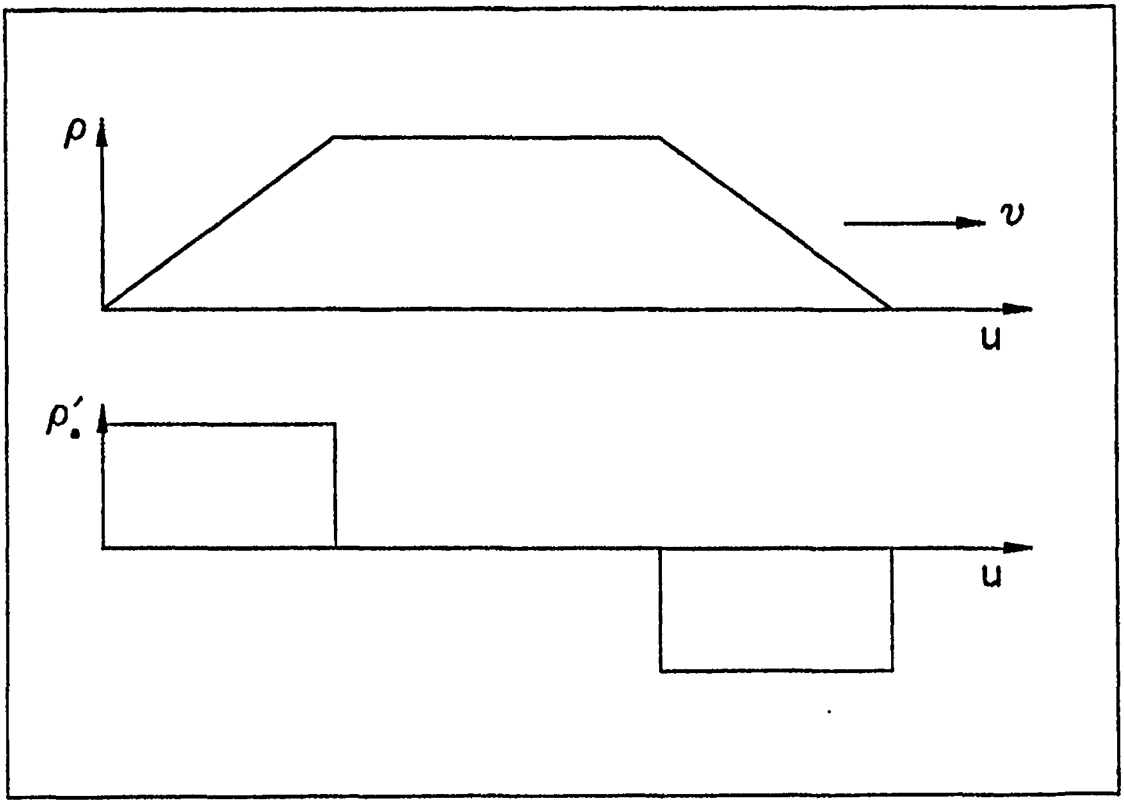

Figure 3. The Functions $\rho_o(u)$ and $\rho_o'(u)$ over the Pulse Lenght

The relationship of the charge density ρ to the function u (Figure 4) must be determined in order to evaluate ρ_o'. This relationship is

$$\rho = -\frac{\rho_o}{\Delta u}u + \rho_o. \tag{2.17}$$

Note that $\Delta u = u_1 - u_2$ (see Figure 1).

Taking the derivative of Equation 2.17 with respect to u, one obtains ρ_m' such that

$$\rho_o'(u) = \rho_m' = -\frac{\rho_o}{\Delta u}. \tag{2.18}$$

Equation 2.18 is now substituted into Equation 2.16 to give

$$B = -n\beta^2\frac{\rho_o}{\Delta u}\int\frac{s}{R^2}dz'. \tag{2.19}$$

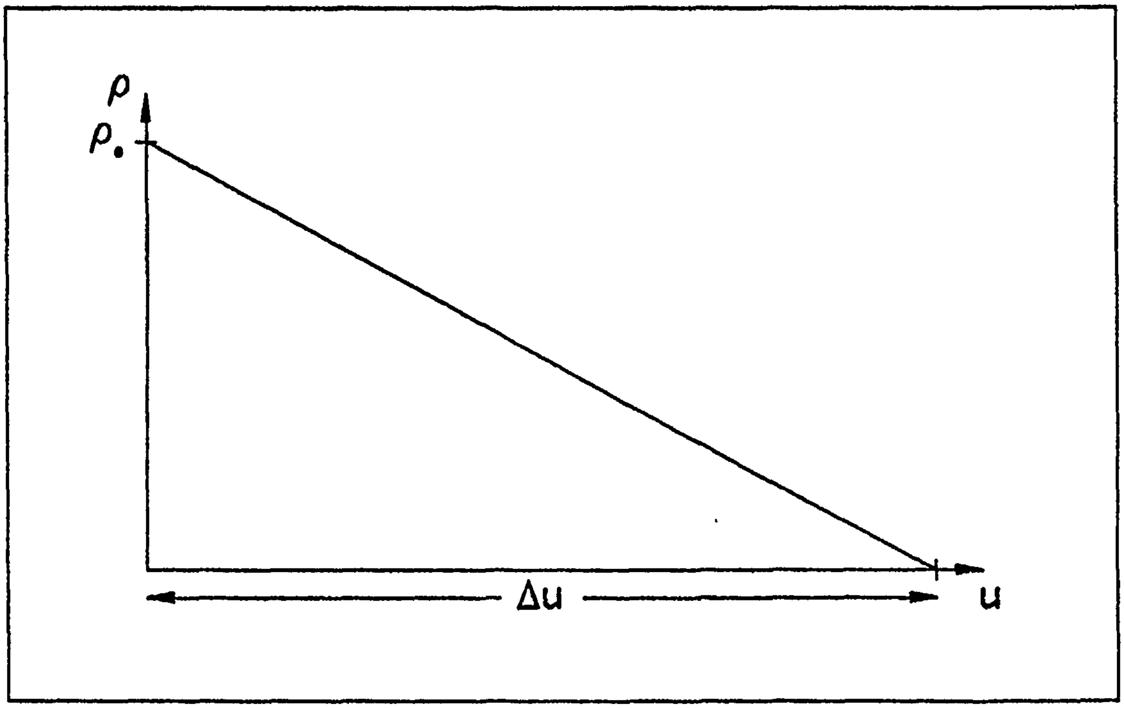

Figure 4. ρ as a Function of u at One End of the Pulse

Since $R^2 = s^2 + (z - z')^2$ and $\Delta u = va$ where a is the rise time, Equation 2.19 can be integrated over the limits of z_i to z_f. Introducing the function $w = z - z'$ and integrating gives the result

$$B = -n\beta^2 \frac{\rho_o}{va} \left\{ \tan^{-1}\left(\frac{w_1}{s}\right) - \tan^{-1}\left(\frac{w_2}{s}\right) \right\}. \tag{2.20}$$

Note that $w_1 = z - z_i'$ and $w_2 = z - z_f'$. Thus, the problem is to find the values of w_1 and w_2. [Ref. 3: p. 14]

C. LIMITS OF INTEGRATION

To find the limits of integration z_i' and z_f' , one must consider two cases. The first case is where the $u(z')$ curve minimum is in the non-zero portion of the $\rho_o'(u)$ function, i.e. between curves t_1 and t_2 of Figure 2. The second case is where the $u(z')$ curve minimum has passed through the non-zero portion of the $\rho_o'(u)$ function, i.e. lower than the curve t_2 of Figure 2.

The first case produces a single integral where the limits of integration are found by solving Equation 2.10 for z'. The function $u(z') = u_1$ where u_1 is the value of the u curve minimum at the upper edge of the $\rho_o'(u)$ function. The result is

$$z_1' = \frac{\{A_1 - \beta'^2 z\} \pm \sqrt{\beta'^2\{(z - A_1)^2 + s^2(1 - \beta'^2)\}}}{(1 - \beta'^2)}. \tag{2.21}$$

Note that $A_1 = u_1 + vt$.

The second case produces two integrals where the limits of integration are also found by solving Equation 2.10 for z'. Two of the limits come from Equation 2.21 while the other two come from where the function $u(z') = u_2$. U_2 is the value of the u curve minimum at the lower edge of the $\rho_o'(u)$ function. The result is Equation 2.21 and

$$z_2' = \frac{\{A_2 - \beta'^2 z\} \pm \sqrt{\beta'^2\{(z - A_2)^2 + s^2(1 - \beta'^2)\}}}{(1 - \beta'^2)}. \tag{2.22}$$

Note that $A_2 = u_2 + vt$.

Another way to describe the limits is that they are the z' values of the intercepts of the $u(z')$ curve with the upper and lower edges of the non-zero part of the $\rho_o'(u)$ function. This is shown in Figure 5.

For the two integrals of the second case, the ordering of the limits is very important. The lower and upper limits of the first integral are $z_1'(\min)$ and $z_2'(\min)$ respectively while the lower and upper limits of the second integral are $z_2'(\max)$ and $z_1'(\max)$ respectively.

D. RISE TIME DEPENDENCE

To derive an approximation for the maximum magnetic field dependence upon rise time, Equation 2.10 must be developed in a power series. Denoting z_o' as the value of z' at which Equation 2.10 has zero slope, the values of z_o', $u(z_o')$, and the second derivative of u with respect to z' [Ref. 2: p. 3751] are

$$z_o' = z - s\left\{\frac{v^2}{c^2} - 1\right\}^{-\frac{1}{2}}, \tag{2.23}$$

$$u(z_o') = z - vt + s\left\{\frac{v^2}{c^2} - 1\right\}^{\frac{1}{2}}, \tag{2.24}$$

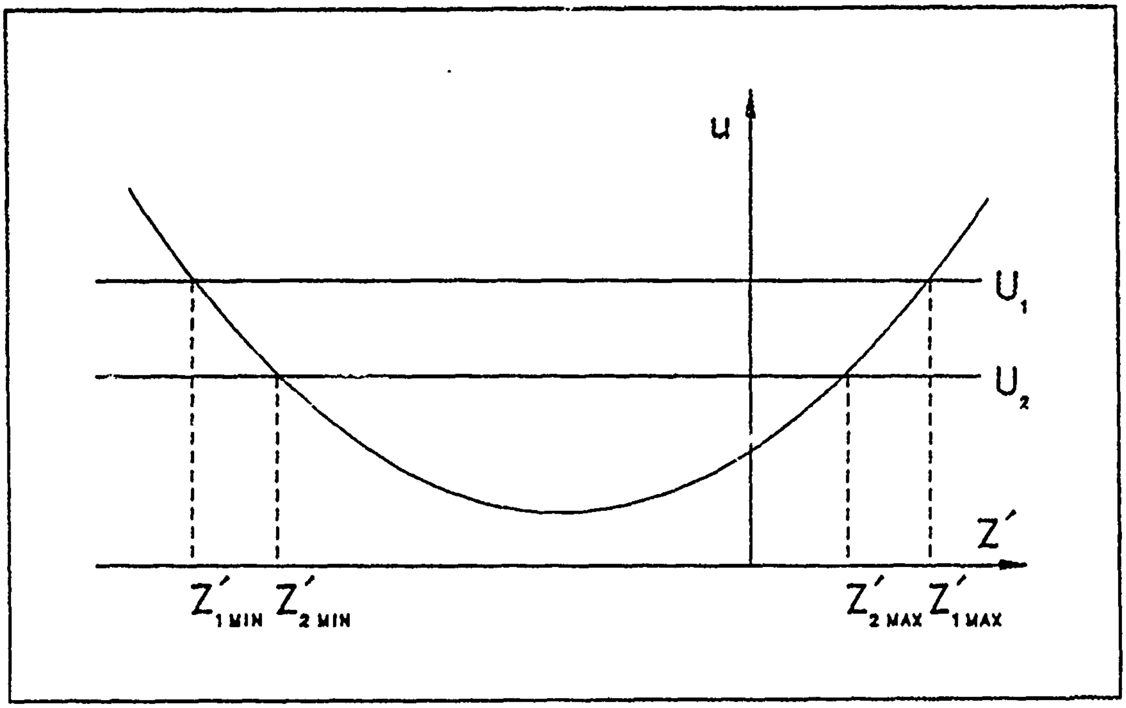

Figure 5. The z' Limits of Integration

$$\frac{\partial^2 u}{\partial z'^2} = \frac{1}{s}\frac{c^2}{\upsilon^2}\left\{\frac{\upsilon^2}{c^2} - 1\right\}^{\frac{3}{2}} \equiv 2D. \tag{2.25}$$

D is a constant defined for conveincnce of use Thus u can now be expressed as a Taylor series about the minimum such that

$$u = u(z_o') + D(z' - z_o')^2. \tag{2.26}$$

The limits z_1' and z_2' can be written in terms of the minimum value as $z_2' = z_o' + \Delta z'$ and $z_1' = z_o' - \Delta z'$ where $\Delta z'$ is the value of $z' - z_o'$. The difference $u(z') - u(z_o') = a$ where a is the rise time (or width) of the current derivative pulse ρ_o' (a can be thought of as a distance by multiplying it by υ). [Ref. 2: p. 3751] Therefore

$$\Delta z' = \left\{\frac{a}{D}\right\}^{\frac{1}{2}}. \tag{2.27}$$

Using this value, the fact that $\Delta u = \upsilon a$, and the assumption that s and R are slowly varying, Equation 2.19 can be evaluated to give

$$B_{max} = n\beta^2 \frac{\rho_o}{\upsilon} \frac{s}{R^2} 2\left\{ \frac{1}{aD} \right\}^{\frac{1}{2}}. \qquad (2.28)$$

Thus the maximum magnetic field has a dependence upon rise time to the negative one-half power. Therefore, as the rise time increases, the maximum magnetic field decreases. [Ref. 2: p. 3752]

Two points should be emphasized. Equation 2.20 gives the exact maximum magnetic field amplitude for w evaluated at time t_2 (see Figure 1). Equation 2.28 gives an approximation for the maximum magnetic field amplitude as a function of the rise time, giving an opportunity to compare and contrast actual behavior.

III. CALCULATIONS AND ANALYSIS

The previous work had identified the exact solution for the maximum magnetic field amplitude and an approximation for the maximum as a function of the rise time. Until this time, no comparison had been conducted between the two to validate the approximation.

Plots of the magnetic field magnitude versus time were generated from evaluating Equation 2.20, using the programs listed in Appendix A. These plots have 200 points each. The time increment between points was adjusted so that the maximum magnetic field magnitude occurred at the fortieth point of the plot. The increment could be so adjusted by knowing that the maximum magnitude would occur at the end of the rise time a. The variables of interest in the programs were the rise time a, the radial distance s, and the axial distance z. Program runs were made for s = 1.0m, 5.0m, and 10.0m; z = 0.0m, 1.0m, and 5.0m; and a = 10.0ps, 20.0ps, and 30.0ps.

Three general trends were observed from these runs. As a increases with s and z fixed, the overall magnetic field magnitude decreases. This is due to the a^{-1} relation in Equation 2.20. As s increases with a and z fixed, the overall magnetic field magnitude decreases. This is due to the s^{-1} relation in the arctangent function of Equation 2.20. And finally, as z increases with a and s fixed, the overall magnetic field magnitude and shape were unchanged except that the pulse was delayed in time more and more as z increased. This is because z is merely a function of time, as seen in Equation 2.10. This time delay behavior can be seen on the horizontal axes of Figure 6, Figure 7, and Figure 8.

To further investigate B_{max}, plots of the maximum magnetic field magnitude versus the rise times associated with each magnitude were generated. This was accomplished by the programs listed in Appendix B. The magnetic field as a function of time was calculated for 200 points by evaluating Equation 2.20, just as did the programs of Appendix A. This was done for a fixed s and a fixed z (z was chosen to be zero for all following calculations since it has no effect on the magnitude and simplifies the calculation process). The initial rise time was selected to be 10.0ps. The magnetic field magnitudes were run through a DO loop to identify the maximum. The maximum, along with the corresponding rise time, was stored in an output data file. The rise time was then incremented by 5.0ps and the entire procedure is repeated three hundred times. The output data file

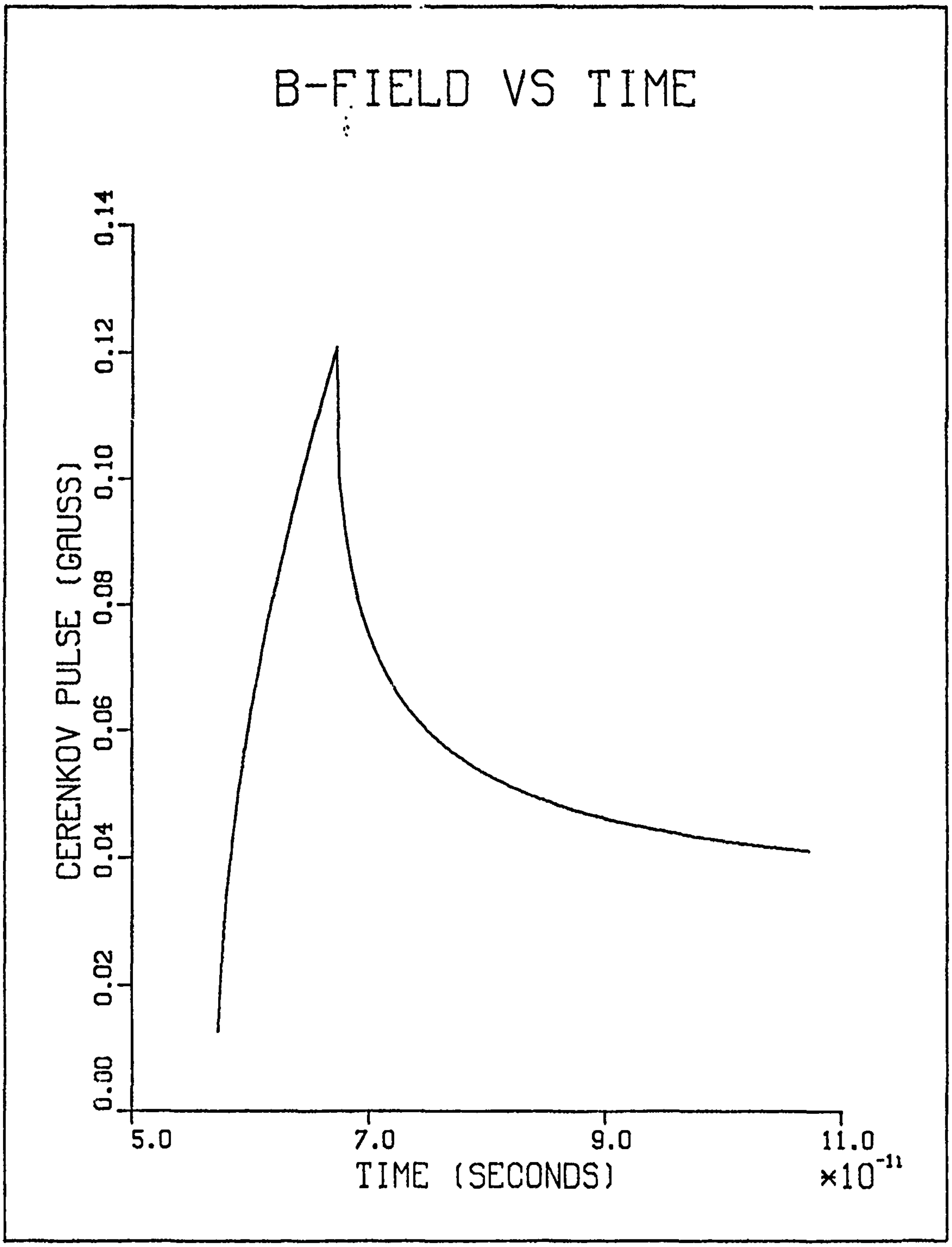

Figure 6. *B* versus *t* at $s = 1.0$m, $z = 0.0$m, and $a = 10.0$ps

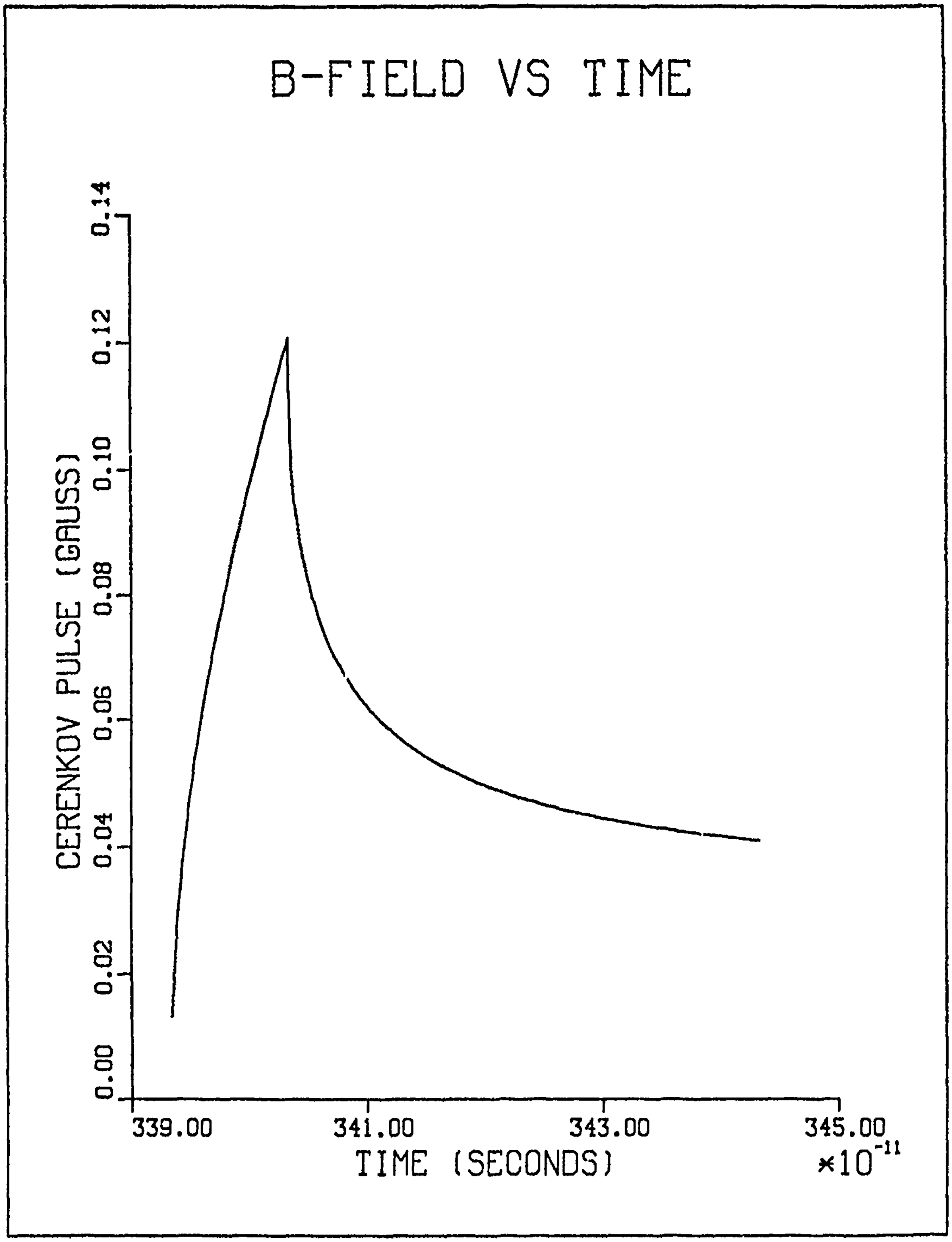

Figure 7. *B* versus t at $s = 1.0$m, $z = 1.0$m, and $a = 10.0$ps

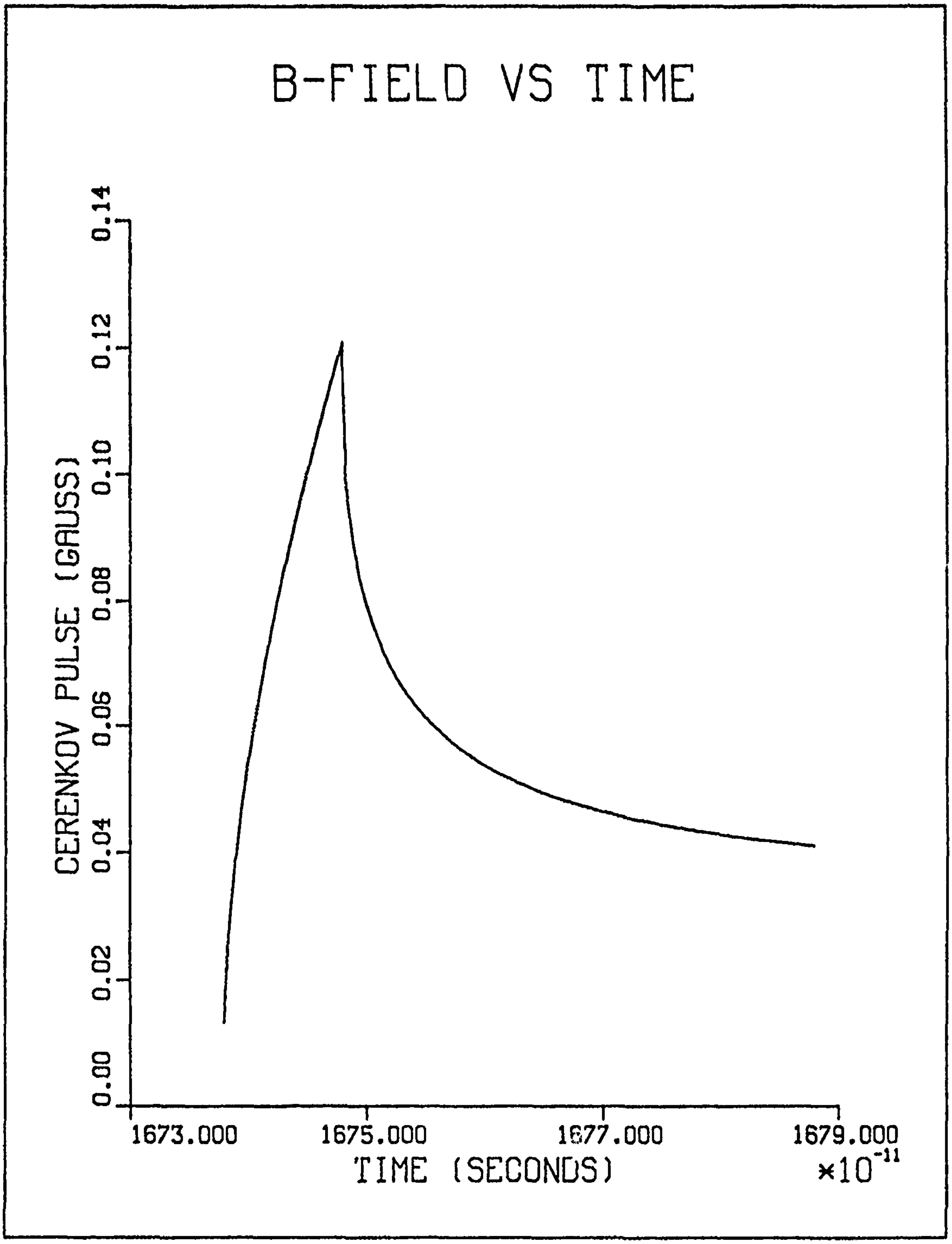

Figure 8. *B* versus *t* at $s = 1.0$m, $z = 5.0$m, and $a = 10.0$ps

of 300 points is plotted by a DISSPLA program incorporated into Appendix B. An example plot can be seen in Figure 9.

The theoretical approximation of Equation 2.28 predicts that the relationship between the maximum magnetic field magnitude B_{max} and the rise time a should be such that $B_{max} \propto a^{-0.5}$. The best way to examine this behavior is to apply a log-log scale to the output data file used to create Figure 9. If the log-log graph produces a linear relationship, the slope of the graph will correspond to the exponent of a. The log-log plot produces a linear relationship close to that expected of Equation 2.28 for low and medium rise times, but the slope begins to decrease as the rise time reaches larger values. This behavior can be seen in Figure 10 and an explanation follows.

The exact solution for the maximum magnetic field magnitude can be calculated by evaluating Equation 2.20 where the limits z' (Equation 2.21) are for the time t_2 (see Figure 1). Since $t_2 = t_1 + a$, z' becomes a function of a. With z arbitrarily chosen to be zero, w/s becomes Ka/s where K is some constant.

The next step is to examine the arctangent function. For large values of Ka/s, the arctangent in Equation 2.20 is approximately constant and large changes in a have little effect on the arctangent value. Therefore the arctangent behaves as if there is no a dependence and B_{max} should be proportional to a^{-1} . But the slope of Figure 10 is tending towards zero for high a values instead of towards -1, the exact opposite of what one might think would happen.

There is one main reason why this behavior occurs. It goes back to the two cases for the limits of integration of Equation 2.20. The maximum magnetic field amplitude was calculated for the single integral case where the limits of integration were evaluated at time t_2 (see Figure 2). The magnetic field amplitude of the "tail" of the pulse was calculated for the double integral case where the limits of integration were evaluated for times $t > t_2$ (see Figure 5).

Since these calculations are for the radiative fields, the magnitude of the magnetic field does not decay to zero but instead eventually decays to some constant value. As the rise time increases for a fixed observation point, the magnitude of the "tail" increases relative to the maximum magnitude of the pulse. Another way to look at the process is that large rise time pulses decay slowly compared to short rise time pulses. For extremely large rise times, the maximum amplitude is virtually indistinguishable from the "tail". Note that the maximum amplitude for a short rise time is larger than the maximum rise time for a long rise time (from theory, $B_{max} \propto a^{-0.5}$). Therefore as the rise time increases, the maximum magnetic field amplitude cannot continue to decrease linearly because the

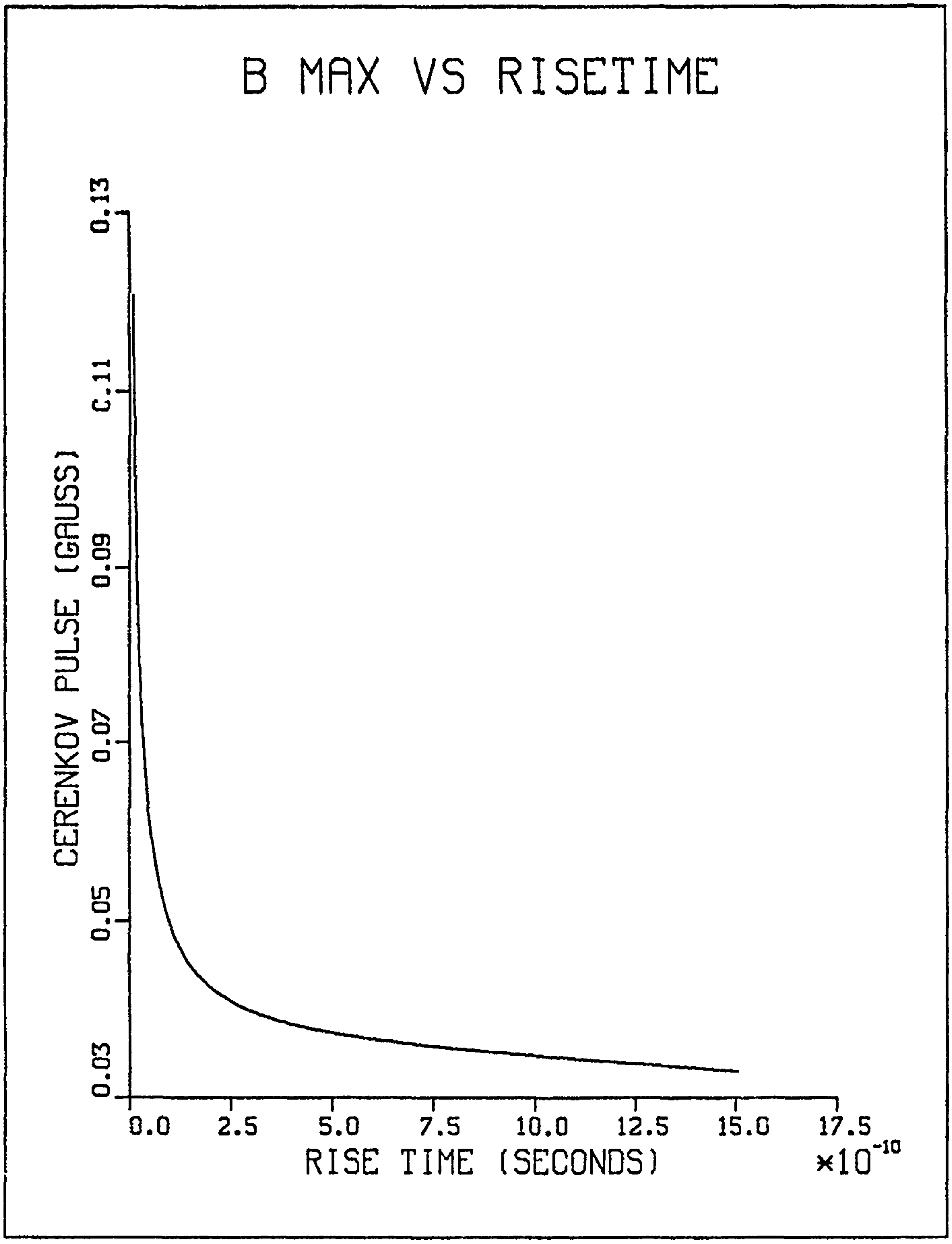

Figure 9. B_{max} versus a at $s = 1.0$m

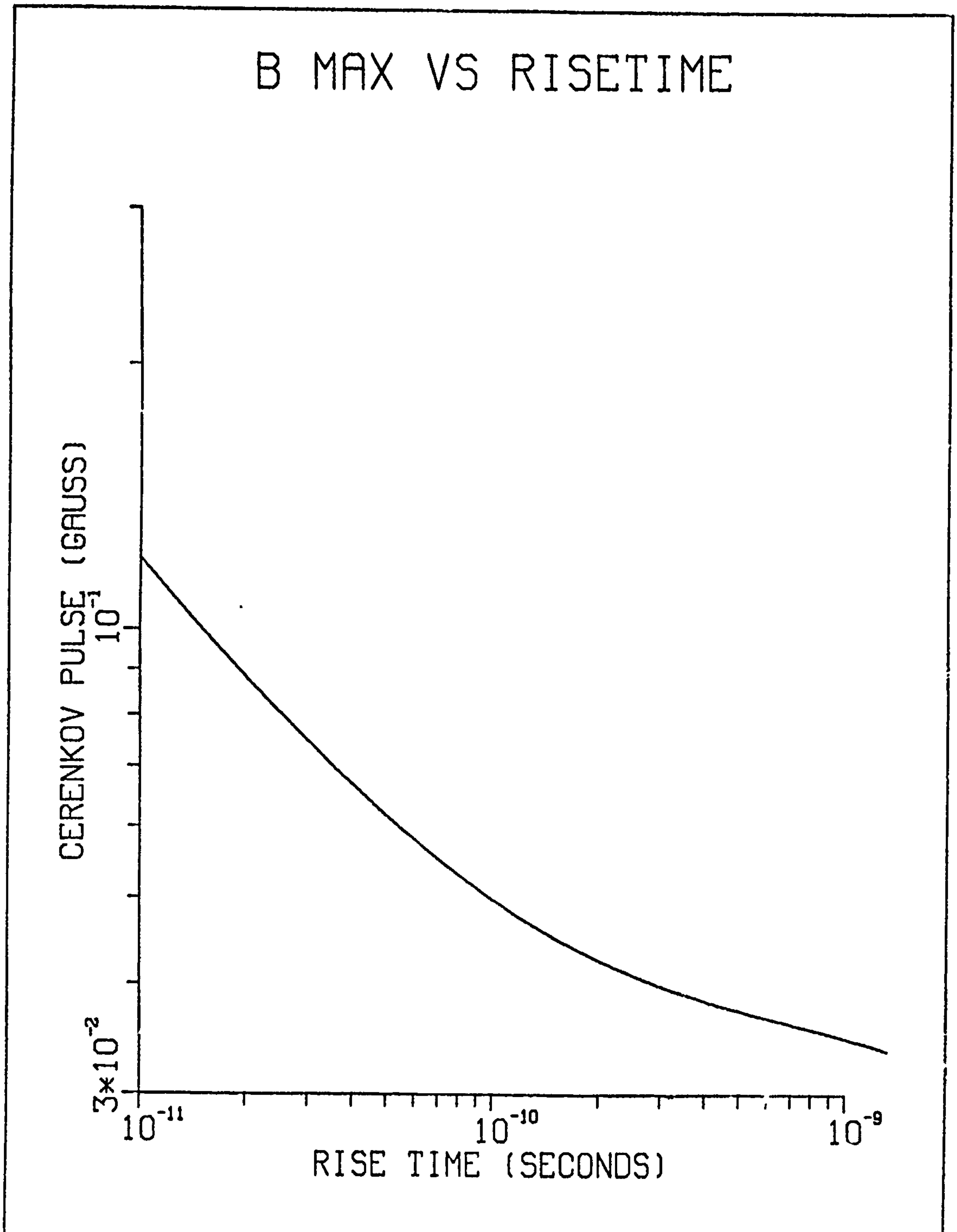

Figure 10. Log B_{max} versus Log a at $s = 1.0$m

maximum amplitude is approaching a constant value as the rise time increases. This behavior is shown in Figure 11 and Figure 12.

But as s increases, it decreases the value of Ka/s and the arctangent leaves the almost constant value range and tends towards the linear response value region. As a result, the relationship between B_{max} and a behaves linearly over a larger range of a as s increases. This behavior can be seen in Figure 13.

The data file used in the plots of Figure 9 and Figure 10 was generated at a fixed s value of 1.0m. To further investigate this behavior, additional data files needed to be calculated and plotted for different values of s to see how much of an effect the radial distance might have on the relationship between B_{max} and a . The results are shown in Table 1.

Table 1. B_{max} VERSUS a SLOPE VALUES

s = 1.00m	slope = -0.112
s = 5.00m	slope = -0.207
s = 10.0m	slope = -0.273
s = 15.0m	slope = -0.358
s = 20.0m	slope = -0.341
s = 25.0m	slope = -0.368
s = 30.0m	slope = -0.409
s = 40.0m	slope = -0.425
s = 50.0m	slope = -0.452
s = 75.0m	slope = -0.467
s = 100.0m	slope = -0.475

The cancelling of the $B_{max} \propto a^{-1}$ effect at high a values by the magnitude of the magnetic pulse decay tail is shown in Figure 14. The effect of reducing the argument of the arctangent function by increasing the radial distance s and thus returning the arctangent function more to its linear response range is shown in Figure 15.

The relationship between the maximum magnetic field magnitude B_{max} and the radial distance s was graphed. The magnetic field was evaluated using Equation 2.20 and was plotted as a function of time for 200 points at a constant s and constant a. The maximum magnetic pulse magnitude is determined and stored with its corresponding s value. The value of s is incremented and the entire procedure repeated until a 300 point data

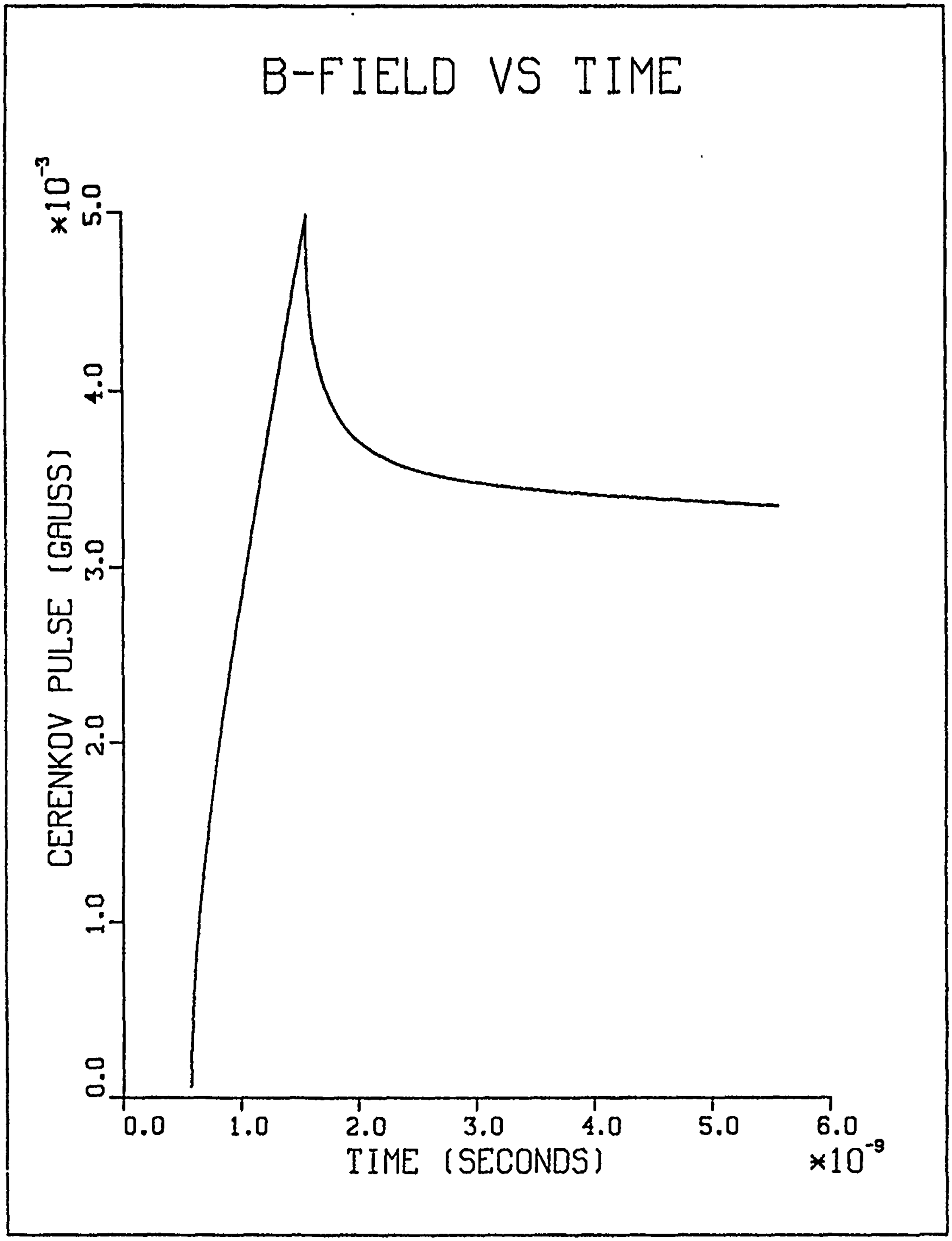

Figure 11. Slow Decay for $a = 1000.0$ps at $s = 10.0$m

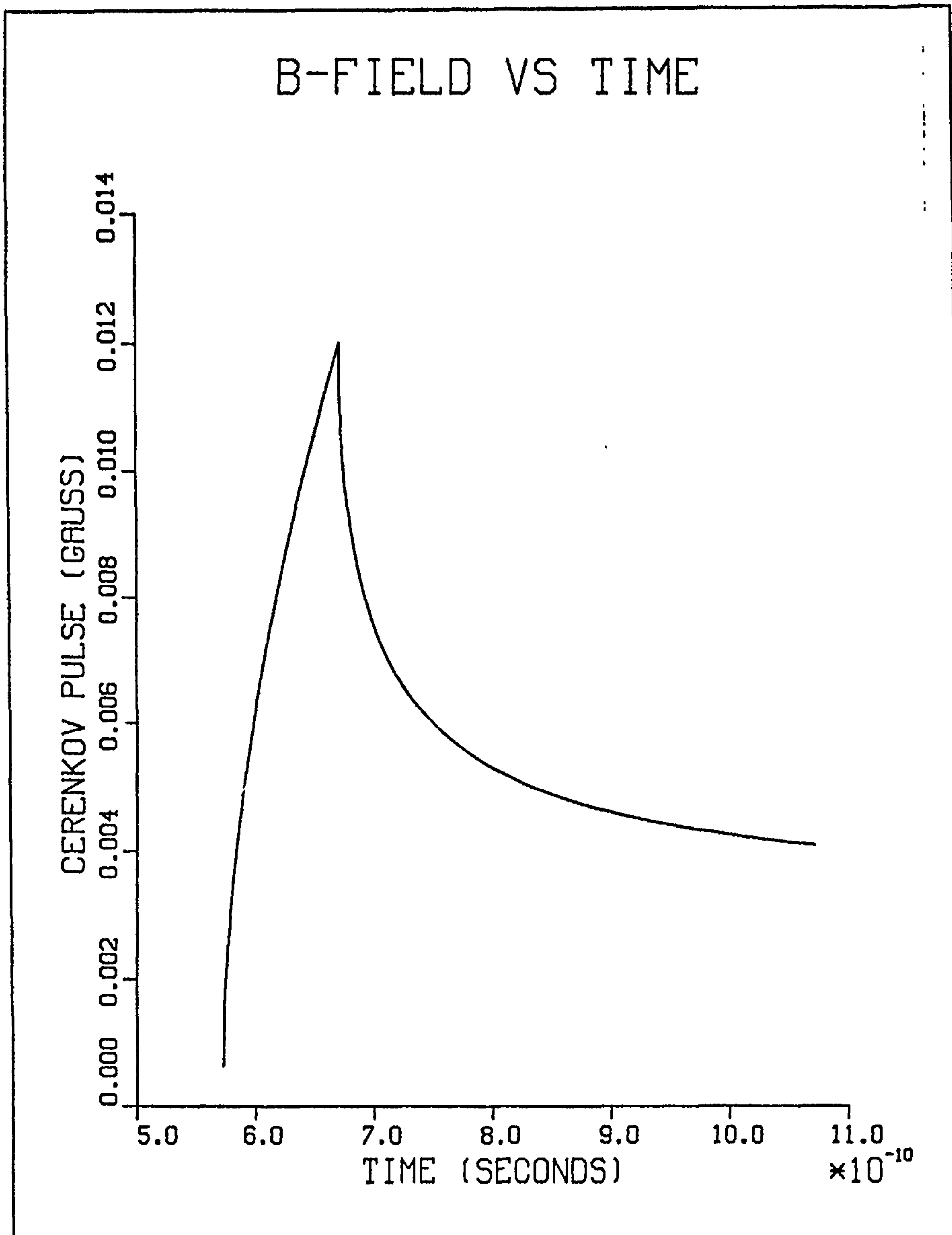

Figure 12. **Fast Decay for $a = 100.0$ps at $s = 10.0$m**

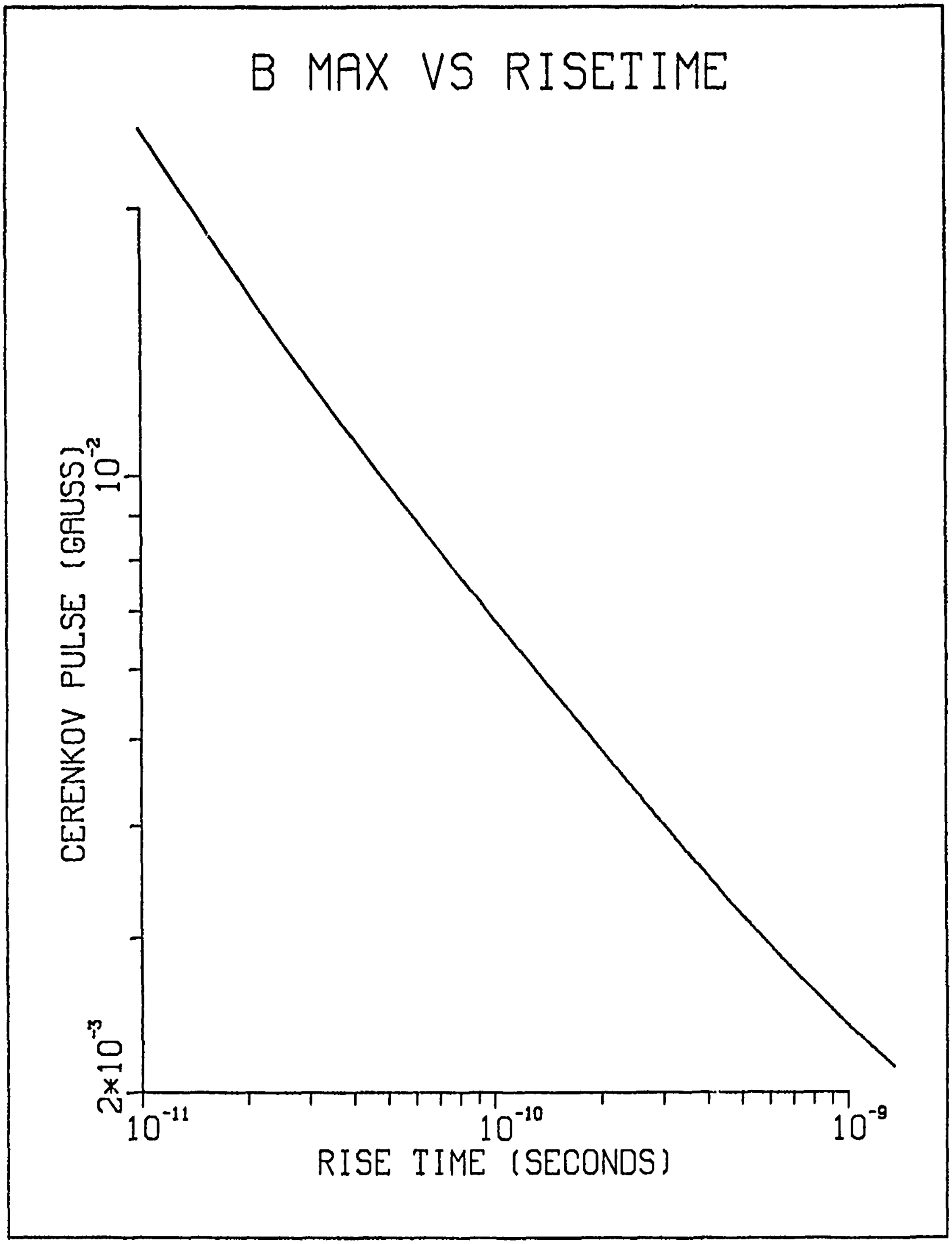

Figure 13. Log B_{max} versus Log a at $s = 30.0$m

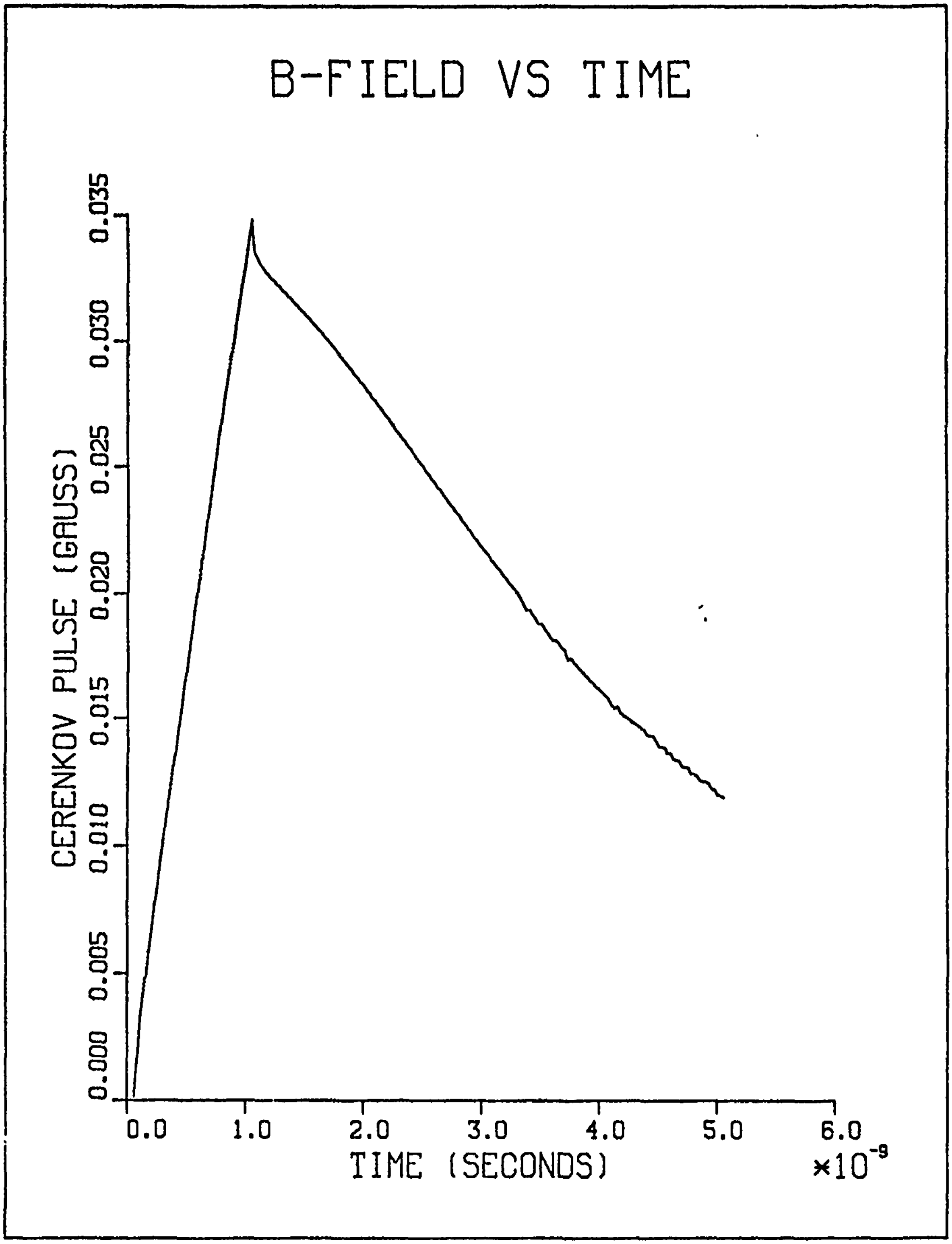

Figure 14. *B* versus *t* at $s = 1.0$m and $a = 1000.0$ps

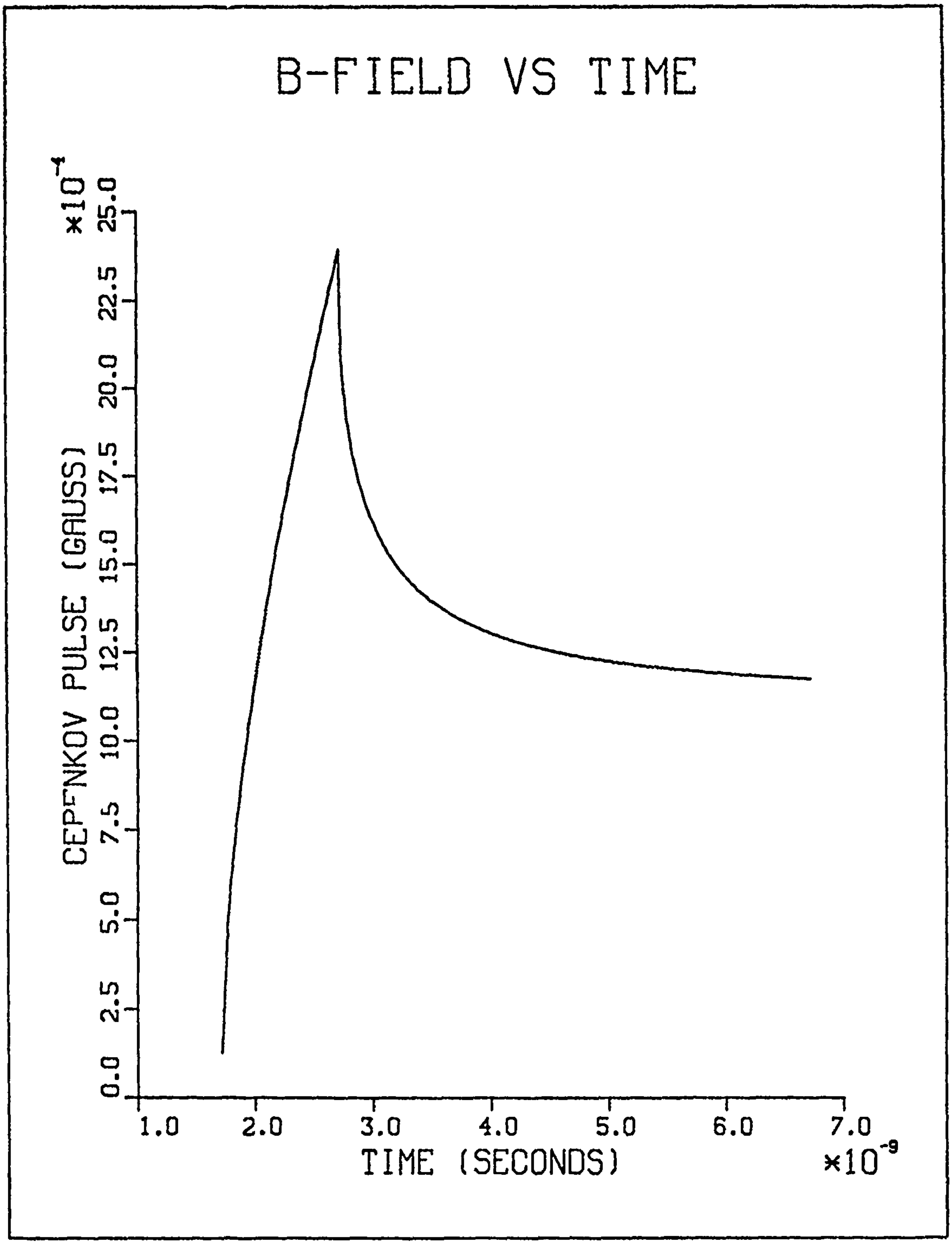

Figure 15. *B* versus *t* at $s = 30.0$m and $a = 1000.0$ps

file has been constructed. This data file is then plotted using DISSPLA. The programs for these actions are located in Appendix C. An example plot can be seen in Figure 16.

The last step is to calculate and plot the three-dimensional relation between time, the magnetic field, and radial distance. The magnetic field is calculated versus time for 50 points using Equation 2.20. The magnitude and corresponding time and radial position are stored in output files. The radial distance is incremented and the entire procedure is repeated 50 times. The result is three output files containing the coordinates for a 50x50 array. A 3-D graphing routine reads these array points, converts them to a surface, and plots the surface. The programs for this procedure are listed in Appendix D.

Due to the fact that DISSPLA will only handle square arrays for surface plotting, the number of points along the $B-t$ axis had to be limited to 50 to hold computer memory requirements within acceptable limits. This somewhat obscures the detail of the 3-D plot. Therefore a limited 2-D projection of $B-t$ curves for a few s values was created in Figure 17 to help understand the workings of Figure 18.

An example plot is displayed in Figure 18. If the graph projection of the $B-s$ plane is rotated about the t axis, a representation of the Cerenkov radiation cone in space is produced.

Several values were used in the programs listed in the appendices. Electron energy was set at 30 MeV. It must be approximately 25 MeV or higher in order for Cerenkov radiation to occur. Current was assumed to be 510 amperes. Any other desired values can be used for these constants. Except for the purposes of Figure 7 and Figure 8, the observer axial distance z was set to zero for simplification of calculations. The observer radial distance s can never be set to zero except for initialization of incrementation use in a DO loop. The function u_1 was set to zero also for simplification. It has the result of causing the function u_2 to simplify to the expression $u_2 = -va$. where a is the rise time. Finally, the magnetic field calculations were conducted in MKS units and converted to CGS units from the relation: 1 Tesla $\approx$ 10,000 Gauss.

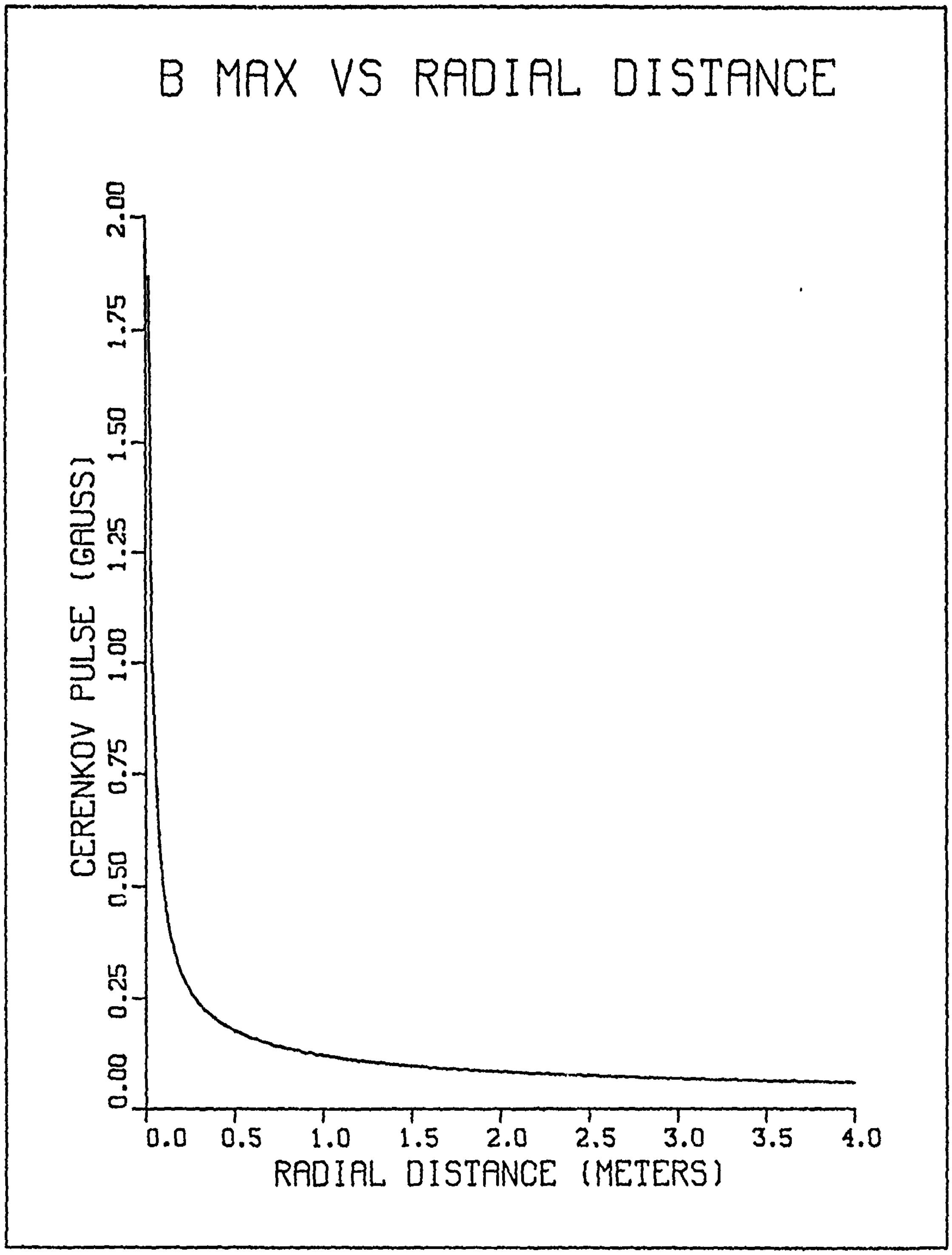

Figure 16. B_{max} **versus** s **at** $a = 10.0$**ps**

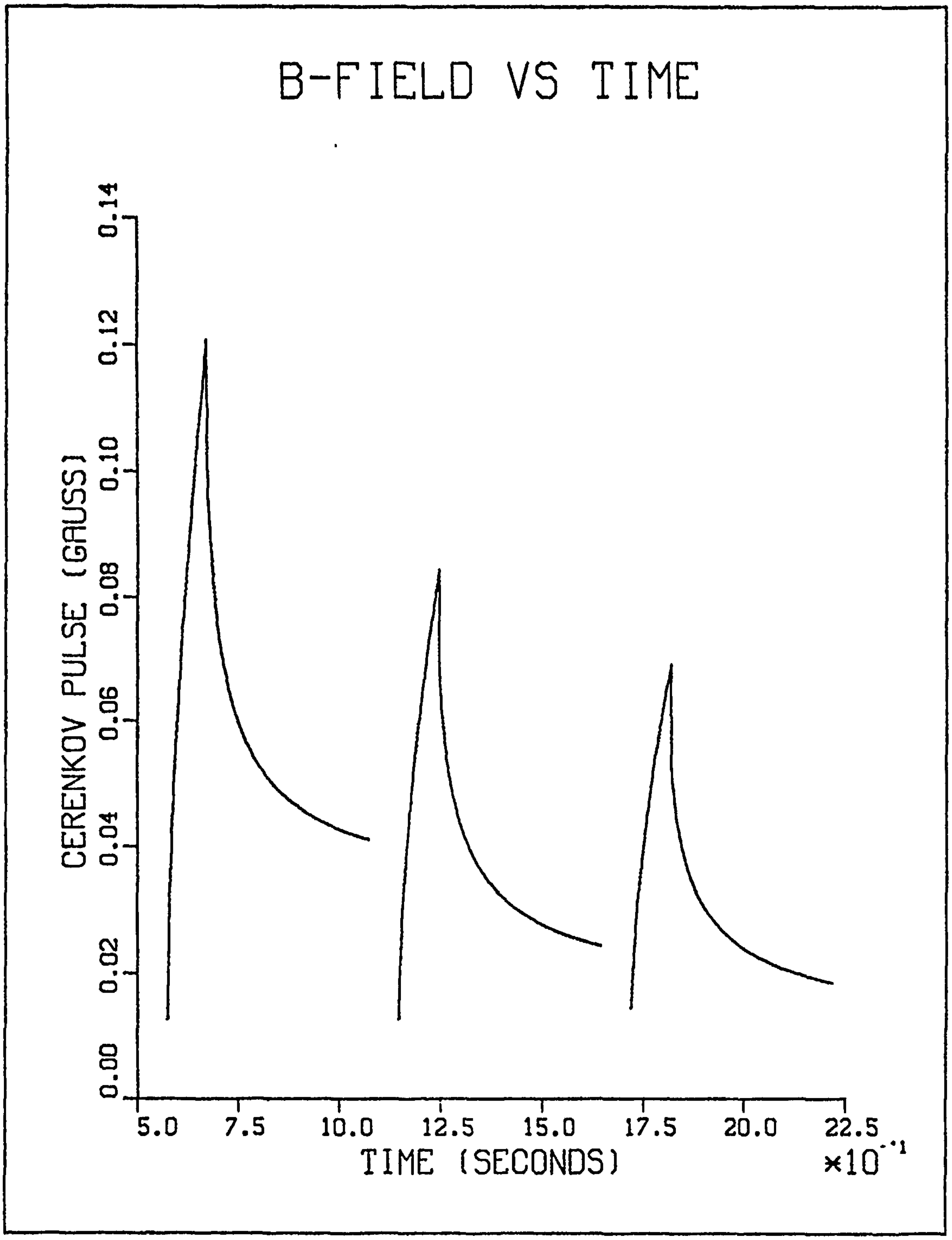

Figure 17. *B* versus *t* for *s* = 1.0m, 2.0m, and 3.0m

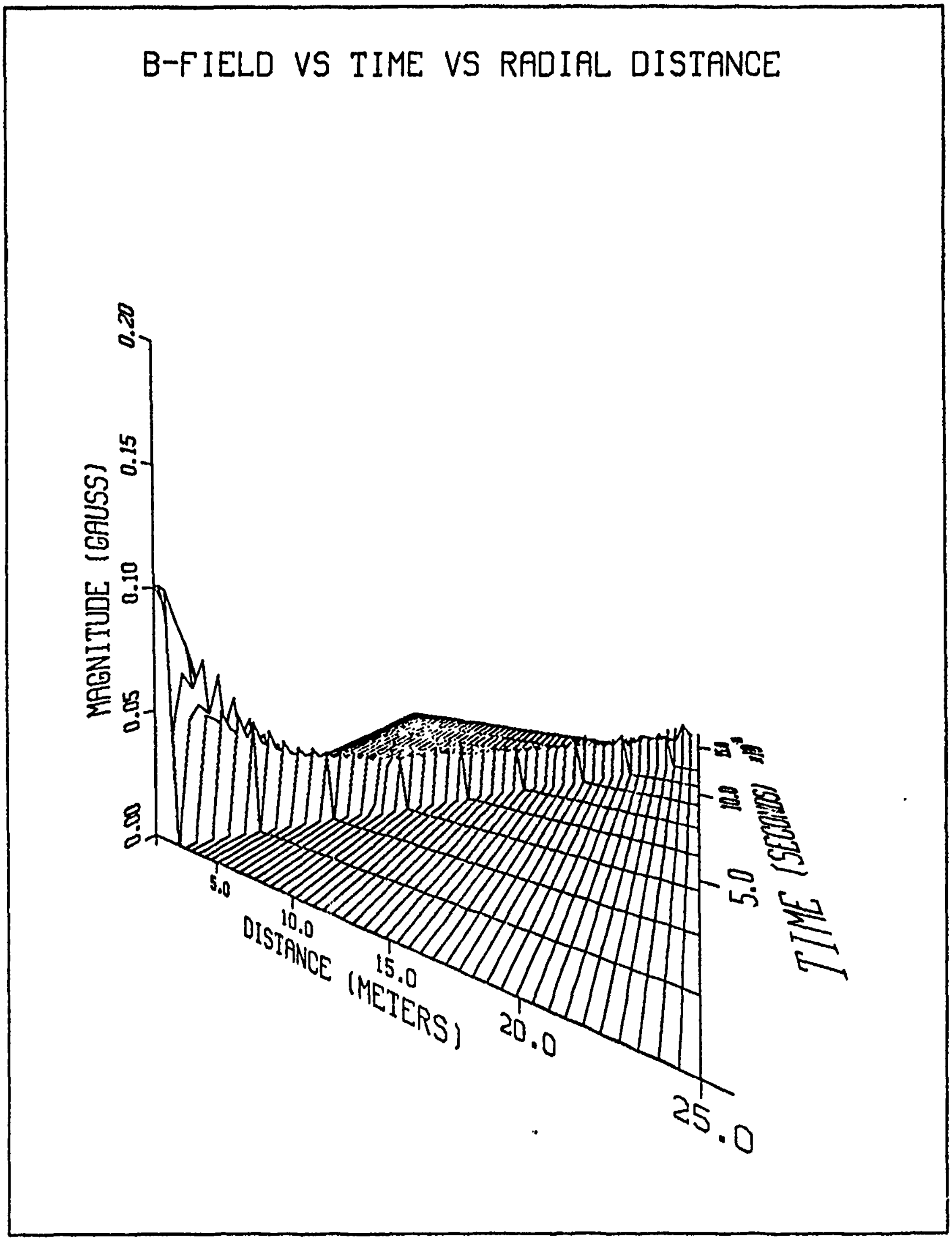

Figure 18. *B* versus *t* versus *s*

IV. CONCLUSIONS AND RECOMMENDATIONS

The primary objective was to determine the effect of varying the rise time of the electron beam charge density upon the Cerenkov pulse magnitude. The theoretical approximation of Equation 2.28 predicts that this behavior should be $B_{max} \propto a^{-0.5}$. Actual results show that this behavior approaches the theoretical value at large radial distances due to the decrease of the argument of the arctangent function.

This study was conducted for the case of a semi-infinite path lenght. Evaluating this behavior for the finite case will have some effect on the expected outcome.

The magnetic field magnitude falls off as the radial distance increases. This was expected from the relationship in Equation 2.20. Since the magnetic field must also travel farther as s increases, the pulse is time-delayed relative to a position closer to the electron beam. By rotating the 3-D graph of Figure 10 about the time axis, an accurate spatial description of the magnetic field pulse is obtained. This figure is the expected Cerenkov radiation cone, somewhat analogous to the wake created by a boat moving faster than the current speed of the water.

The shape and magnitude of the Cerenkov pulse is not affected in any manner by changes in the observer's axial position. It is only affected by changes in the observer's radial position. This assumes that all other possible variables (energy, rise time, current, etc.) are held constant. The tail of such a pulse decays to some constant value which is not zero. This effect is because the calculations are for the radiative fields. Calculations should be conducted with the non-radiative fields as well.

APPENDIX A. CERENKOV PULSE PROGRAM

```
      PROGRAM PULSEI
C
C     ***********************************************************************
C     * THIS PROGRAM IS LOCATED ON THE NPS MAIN FRAME COMPUTER AND IS      *
C     * WRITTEN USING WATFOR 77. AFTER ENTERING CERTAIN PARAMETERS, THE    *
C     * SHAPE OF THE CERENKOV PULSE IS GENERATED. THE VALUES OF THE        *
C     * PULSE MAGNITUDE POINTS FOR THE GIVEN PARAMETERS ARE STORED IN      *
C     * AN OUTPUT FILE FOR LATER USE IN A PLOTTING ROUTINE.                *
C     ***********************************************************************
C
      REAL N, U1, U2, BETA, CO, ROE, A1, A2, BPRME
      REAL RISETM, D, DD, E1, E2, Z1I, Z1F, Z2I, Z2F
      REAL B1, B2, W1, W2, YY, XX, T1, T2, WPI, WPF
      REAL ZPMIN, SSQ, BPSQ, TINC, IAMP, ENER, ENERJ
      REAL S, Z, C, MO, V, NEG, TPRME, B
      INTEGER I
      DIMENSION TPRME(200), B(200)
      DATA TPRME/200*0.0/, B/200*0.0/
C
      OPEN (UNIT=4,FILE='TERMINAL')
      OPEN (UNIT=7,FILE='PULPLT DATA A')
C
C     ***********************************************************************
C     * N IS THE INDEX OF REFRACTION. IAMP IS THE ACCELERATOR CURRENT      *
C     * IN AMPS. ENER IS THE ELECTRON ENERGY IN MEV. RISETM IS THE         *
C     * PULSE RISETIME IN SECONDS. S IS THE OBSERVER RADIAL DISTANCE IN    *
C     * METERS. Z IS THE OBSERVER AXIAL DISTANCE IN METERS. CO IS THE      *
C     * VACUUM SPEED OF LIGHT IN METERS PER SECONDS. MO IS THE ELECTRON    *
C     * MASS IN KILOGRAMS. ROE IS THE LINEAR CHARGE DENSITY IN COULOMBS    *
C     * PER METER.                                                         *
C     ***********************************************************************
C
      N = 1.000293
      IAMP = 510.0
      ENER = 30.0
      RISETM = 10.0E-12
      S = 1.0
      Z = 5.0
C
      CO = 2.99792458E08
      MO = 9.109534E-31
      C = CO/N
      ENERJ = ENER*1.6021892E-13
      V = CO*SQRT(1.0-((MO**2.0)*(CO**4.0)/(ENERJ**2.0)))
      IF (V.LE.C) GO TO 60
      BETA = V/CO
      ROE = IAMP/V
      BPRME = N*BETA
C
C     ***********************************************************************
C     * SETTING U1 ARBITRARILY TO ZERO FIXES U2. THIS IS TO SIMPLIFY       *
```

```
C     * CALCULATIONS.                                                       *
C     ***********************************************************************
C
        U1 = 0.0
        U2 = -V*RISETM
C
C     ***********************************************************************
C     * WITH 200 POINTS, TINC WAS SELECTED SO AS FOR THE PULSE MAXIMUM  *
C     * TO OCCUR AT APPROXIMATELY 20% OF THE GRAPH LENGHT.              *
C     ***********************************************************************
C
        TINC = RISETM/40.0
        SSQ = S**2.0
        BPSQ = BPRME**2.0
        D = Z*BPSQ
        DD = 1 - BPSQ
        NEG = SSQ*DD
C
C     ***********************************************************************
C     * ZPMIN IS THE Z VALUE OF THE MINIMUM OF THE U CURVE.                *
C     ***********************************************************************
C
        ZPMIN = Z - (S/(SQRT(BPSQ - 1.0)))
C
C     ***********************************************************************
C     * T1 AND T2 ARE THE TIMES THAT THE U CURVE MINIMUM IS TANGENT TO  *
C     * THE U1 AND U2 LEVELS RESPECTIVELY.                               *
C     ***********************************************************************
C
        T1 = (ZPMIN - U1)/V + (SQRT(SSQ + (Z - ZPMIN)**2.0))/C
        T2 = T1 + RISETM
C
        DO 40 I = 1,200
           IF (I.EQ.1) THEN
              TPRME(I) = T1
              B(I) = 0.0
           ELSE
              TPRME(I) = T1 + (REAL(I)*TINC)
           END IF
C
C     ***********************************************************************
C     * THIS IF STATEMENT HANDLES THE CHANGEOVER IN LIMITS OF INTEGRAT- *
C     * ION FROM ONE INTERVAL TO TWO INTERVALS.                          *
C     ***********************************************************************
C
           IF (TPRME(I).GT.T2) GO TO 10
           A1 = U1 + V*TPRME(I)
           E1 = (Z - A1)**2.0
C
C     ***********************************************************************
C     * THIS IF STATEMENT HANDLES THE CASE OF HAVING A NEGATIVE SQUARE  *
C     * ROOT IN REAL SPACE WHEN DETERMINING THE LIMITS OF INTEGRATION.  *
C     ***********************************************************************
C
           IF (E1.LE.ABS(NEG)) THEN
              B(I) = 0.0
```

```
         GO TO 30
      ELSE
         Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
         Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
      ENDIF
C
      WPI = Z - Z1I
      WPF = Z - Z1F
      W1 = WPI/S
      W2 = WPF/S
      YY = ATAN(W1)
      XX = ATAN(W2)
      B(I) = -ROE*N*(BETA**2.0)*(YY-XX)*1.0E04/U2
      GO TO 20
C
   10 CONTINUE
      A1 = U1 + V*TPRME(I)
      A2 = U2 + V*TPRME(I)
      E1 = (Z - A1)**2.0
      E2 = (Z - A2)**2.0
      IF (E1.LE.ABS(NEG)) THEN
         B(I) = 0.0
         GO TO 30
      ELSE
         Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
         Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
      ENDIF
C
      IF (E2.LE.ABS(NEG)) THEN
         B(I) = 0.0
         GO TO 30
      ELSE
         Z2I = ((A2-D) + SQRT(BPSQ*(E2+NEG)))/DD
         Z2F = ((A2-D) - SQRT(BPSQ*(E2+NEG)))/DD
      ENDIF
C
      WPI = Z - Z1I
      WPF = Z - Z2I
      W1 = WPI/S
      W2 = WPF/S
      YY = ATAN(W1)
      XX = ATAN(W2)
      B1 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
C
      WPI = Z - Z2F
      WPF = Z - Z1F
      W1 = WPI/S
      W2 = WPF/S
      YY = ATAN(W1)
      XX = ATAN(W2)
      B2 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
      B(I) = B1 + B2
C
   20 CONTINUE
C
```

```
   30    CONTINUE
C
   40 CONTINUE
C
      DO 50 I = 1,200
         WRITE(7,1000) TPRME(I),B(I)
 1000    FORMAT(G15.7,G15.7)
   50 CONTINUE
C
      GO TO 70
   60 WRITE(4,1010)
 1010 FORMAT(/,'V IS LESS THAN C, THEREFORE THERE IS NO CERENKOV RADIATI
     $ON FOR THIS ELECTRON',4X,' ENERGY')
   70 CONTINUE
C
      STOP
      END
C
C
C
      PROGRAM PULPLT
C
      REAL X(300),Y(300),XMIN,XMAX,YMIN,YMAX
      INTEGER N
      CHARACTER*20 TITLE$
C
      TITLE$ = 'B-FIELD VS TIME$'
      N = 200
C
      READ(7,8000) ((X(I),Y(I)),I=1,N)
 8000 FORMAT(2G15.7)
C
C     CALL TEK618
      CALL SHERPA('P6      ','B',3)
      CALL PAGE(6,7.5)
      CALL AREA2D(4.5,5.5)
      CALL XNAME('TIME (SECONDS)$',100)
      CALL YNAME('CERENKOV PULSE (GAUSS)$',100)
      CALL HEADIN(TITLE$,100,1.5,1)
      CALL CROSS
      CALL RANGE(X,XMIN,XMAX,N)
      CALL RANGE1(Y,YMIN,YMAX,N)
      CALL GRAF(XMIN,'SCALE',XMAX,YMIN,'SCALE',YMAX)
      CALL CURVE(X,Y,200,0)
      CALL ENDPL(0)
      CALL DONEPL
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
```

```
      YMAX=-1.E20
      DO 10 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   10 CONTINUE
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE1(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
      YMAX=-1.E20
      DO 20 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   20 CONTINUE
C
      RETURN
      END
```

APPENDIX B. B MAX - RISE TIME PROGRAM

```
      PROGRAM CERENKV
C
C     ***********************************************************************
C     * THIS PROGRAM IS LOCATED ON THE NPS MAIN FRAME COMPUTER AND IS       *
C     * WRITTEN USING WATFOR 77. AFTER ENTERING CERTAIN PARAMETERS, THE     *
C     * SHAPE OF THE CERENKOV PULSE IS GENERATED. THE MAXIMUM VALUE OF      *
C     * THE PULSE MAGNITUDE IS DETERMINED AND STORED WITH ITS CORRES-       *
C     * PONDIND RISE TIME VALUE. THE RISE TIME IS INCREMENTED AND THE       *
C     * ENTIRE PROCESS IS REPEATED. THESE VALUES ARE THEN READ TO AN        *
C     * OUTPUT FILE FOR USE IN A PLOTTING ROUTINE.
C     ***********************************************************************
C
      REAL N, U1, U2, BETA, CO, ROE, A1, A2, BPRME, YMAX
      REAL RISETM, D, DD, E1, E2, Z1I, Z1F, Z2I, Z2F
      REAL B1, B2, W1, W2, YY, XX, T1, T2, WPI, WPF
      REAL ZPMIN, SSQ, BPSQ, TINC, IAMP, ENER, ENERJ
      REAL S, Z, C, MO, V, NEG, TPRME, B, DRISE, BMAX
      INTEGER I, J
      DIMENSION TPRME(200), B(200), DRISE(300), BMAX(300)
      DATA TPRME/200*0.0/, B/200*0.0/, DRISE/300*0.0/, BMAX/300*0.0/
C
      OPEN (UNIT=7,FILE='PULPLT DATA A')
      OPEN (UNIT=4,FILE='TERMINAL')
      OPEN (UNIT=7,FILE='CPLOT1 DATA A')
C
C     ***********************************************************************
C     * N IS THE INDEX OF REFRACTION. IAMP IS THE ACCELERATOR CURRENT       *
C     * IN AMPS. ENER IS THE ELECTRON ENERGY IN MEV. RISETM IS THE          *
C     * PULSE RISETIME IN SECONDS. S IS THE OBSERVER RADIAL DISTANCE IN     *
C     * METERS. Z IS THE OBSERVER AXIAL DISTANCE IN METERS. CO IS THE       *
C     * VACUUM SPEED OF LIGHT IN METERS PER SECONDS. MO IS THE ELECTRON     *
C     * MASS IN KILOGRAMS. ROE IS THE LINEAR CHARGE DENSITY IN COULOMBS     *
C     * PER METER.                                                          *
C     ***********************************************************************
C
      N = 1.000293
      IAMP = 510.0
      ENER = 30.0
      RISETM = 9.9E-12
      S = 10.0
      Z = 0.0
C
      CO = 2.99792458E08
      MO = 9.109534E-31
      C = CO/N
      ENERJ = ENER*1.6021892E-13
      V = CO*SQRT(1.0-((MO**2.0)*(CO**4.0)/(ENERJ**2.0)))
      IF (V.LE.C) GO TO 80
      BETA = V/CO
      ROE = IAMP/V
```

```
      BPRME = N*BETA
C
C    **********************************************************************
C    * SETTING U1 ARBITRARILY TO ZERO FIXES U2. THIS IS TO SIMPLIFY     *
C    * CALCULATIONS.                                                      *
C    **********************************************************************
C
      U1 = 0.0
C
C    **********************************************************************
C    * WITH 200 POINTS, TINC WAS SELECTED SO AS FOR THE PULSE MAXIMUM   *
C    * TO OCCUR AT APPROXIMATELY 20% OF THE GRAPH LENGHT.                 *
C    **********************************************************************
C
      SSQ = S**2.0
      BPSQ = BPRME**2.0
      D = Z*BPSQ
      DD = 1 - BPSQ
      NEG = SSQ*DD
C
C    **********************************************************************
C    * ZPMIN IS THE Z VALUE OF THE MINIMUM OF THE U CURVE.                *
C    **********************************************************************
C
      ZPMIN = Z - (S/(SQRT(BPSQ - 1.0)))
C
C    **********************************************************************
C    * T1 AND T2 ARE THE TIMES THAT THE U CURVE MINIMUM IS TANGENT TO   *
C    * THE U1 AND U2 LEVELS RESPECTIVELY. SINCE T2 AND U2 ARE FUNCT-    *
C    * IONS OF RISETM, THEY ARE INSIDE THE DO LOOP.                       *
C    **********************************************************************
C
      T1 = (ZPMIN - U1)/V + (SQRT(SSQ + (Z - ZPMIN)**2.0))/C
C
      DO 60 J = 1,300
         RISETM = RISETM + 1.0E-13
         TINC = RISETM/40.0
         DRISE(J) = RISETM
         U2 = -V*RISETM
         T2 = T1 + RISETM
C
         DO 40 I = 1,200
            IF (I.EQ.1) THEN
               TPRME(I) = T1
               B(I) = 0.0
            ELSE
               TPRME(I) = T1 + (REAL(I)*TINC)
            END IF
C
C    **********************************************************************
C    * THIS IF STATEMENT HANDLES THE CHANGEOVER IN LIMITS OF INTEGRAT- *
C    * ION FROM ONE INTERVAL TO TWO INTERVALS.                            *
C    **********************************************************************
C
            IF (TPRME(I).GT.T2) GO TO 10
            A1 = U1 + V*TPRME(I)
```

```
         E1 = (Z - A1)**2.0
C
C   ****************************************************************************
C   * THIS IF STATEMENT HANDLES THE CASE OF HAVING A NEGATIVE SQUARE   *
C   * ROOT IN REAL SPACE WHEN DETERMINING THE LIMITS OF INTEGRATION.   *
C   ****************************************************************************
C
         IF (E1.LE.ABS(NEG)) THEN
            B(I) = 0.0
            GO TO 30
         ELSE
            Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
            Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
         ENDIF
C
         WPI = Z - Z1I
         WPF = Z - Z1F
         W1 = WPI/S
         W2 = WPF/S
         YY = ATAN(W1)
         XX = ATAN(W2)
         B(I) = -ROE*N*(BETA**2.0)*(YY-XX)*1.0E04/U2
         GO TO 20
C
   10    CONTINUE
         A1 = U1 + V*TPRME(I)
         A2 = U2 + V*TPRME(I)
         E1 = (Z - A1)**2.0
         E2 = (Z - A2)**2.0
         IF (E1.LE.ABS(NEG)) THEN
            B(I) = 0.0
            GO TO 30
         ELSE
            Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
            Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
         ENDIF
C
         IF (E2.LE.ABS(NEG)) THEN
            B(I) = 0.0
            GO TO 30
         ELSE
            Z2I = ((A2-D) + SQRT(BPSQ*(E2+NEG)))/DD
            Z2F = ((A2-D) - SQRT(BPSQ*(E2+NEG)))/DD
         ENDIF
C
         WPI = Z - Z1I
         WPF = Z - Z2I
         W1 = WPI/S
         W2 = WPF/S
         YY = ATAN(W1)
         XX = ATAN(W2)
         B1 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
C
         WPI = Z - Z2F
         WPF = Z - Z1F
         W1 = WPI/S
```

```
              W2 = WPF/S
              YY = ATAN(W1)
              XX = ATAN(W2)
              B2 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
              B(I) = B1 + B2
C
   20         CONTINUE
C
   30         CONTINUE
C
   40      CONTINUE
C
           YMAX = -1.0E20
C
C    **********************************************************************
C    * THIS LOOP DETERMINES THE MAXIMUM VALUE OF THE CERENKOV PULSE.     *
C    **********************************************************************
C
           DO 50 I = 1,200
              IF (B(I).GT.YMAX) YMAX = B(I)
   50      CONTINUE
C
           BMAX(J) = YMAX
   60 CONTINUE
C
      DO 70 I = 1,300
         WRITE(7,1000) DRISE(I),BMAX(I)
 1000    FORMAT(G15.7,G15.7)
   70 CONTINUE
C
      GO TO 90
   80 WRITE(4,1010)
 1010 FORMAT(/,'V IS LESS THAN C, THEREFORE THERE IS NO CERENKOV RADIATI
     $ON FOR THIS ELECTRON',4X,' ENERGY')
   90 CONTINUE
C
      STOP
      END
C
C
C
      PROGRAM CPLOT1
C     PROGRAM LOGPLT
C
      REAL X(300),Y(300),XMIN,XMAX,YMIN,YMAX
C     DOUBLE PRECISION X(300),Y(300),XMIN,XMAX,YMIN,YMAX
      INTEGER N
      CHARACTER*20 TITLE$
C
      TITLE$ = 'B MAX VS RISETIME$'
      N = 300
C
      READ(7,8000) ((X(I),Y(I)),I=1,N)
 8000 FORMAT(2G15.7)
C
C     CALL TEK618
```

```
      CALL SHERPA('P7      ','B',3)
C     CALL SHERPA('P8      ','B',3)
      CALL PAGE(6,7.5)
      CALL AREA2D(4.5,5.5)
      CALL XNAME('RISE TIME (SECONDS)$',100)
      CALL YNAME('CERENKOV PULSE (GAUSS)$',100)
      CALL HEADIN(TITLE$,100,1.5,1)
      CALL CROSS
      CALL RANGE(X,XMIN,XMAX,N)
      CALL RANGE1(Y,YMIN,YMAX,N)
      CALL GRAF(XMIN,'SCALE',XMAX,YMIN,'SCALE',YMAX)
C     CALL LOGLOG(XMIN,4.5,YMIN,5.5)
      CALL CURVE(X,Y,300,0)
      CALL ENDPL(0)
      CALL DONEPL
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
      YMAX=-1.E20
      DO 10 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   10 CONTINUE
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE1(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
      YMAX=-1.E20
      DO 20 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   20 CONTINUE
C
      RETURN
      END
```

APPENDIX C. B MAX - S PROGRAM

```
      PROGRAM BVS
C
C    ************************************************************************
C    * THIS PROGRAM IS LOCATED ON THE NPS MAIN FRAME COMPUTER AND IS        *
C    * WRITTEN USING WATFOR 77. AFTER ENTERING CERTAIN PARAMETERS, THE      *
C    * SHAPE OF THE CERENKOV PULSE IS GENERATED. THE MAXIMUM OF THE         *
C    * PULSE MAGNITUDE IS DETERMINED AND STORED WITH ITS CORRESPONDING      *
C    * S VALUE. THE S VALUE IS THEN INCREMENTED AND THE ENTIRE PROCESS      *
C    * IS REPEATED. THESE VALUES ARE READ TO AN OUTPUT FILE FOR LATER       *
C    * USE IN A PLOTTING ROUTINE.                                           *
C    ************************************************************************
C
      REAL N, U1, U2, BETA, CO, ROE, A1, A2, BPRME, YMAX
      REAL RISETM, D, DD, E1, E2, Z1I, Z1F, Z2I, Z2F
      REAL B1, B2, W1, W2, YY, XX, T1, T2, WPI, WPF
      REAL ZPMIN, SSQ, BPSQ, TINC, IAMP, ENER, ENERJ
      REAL S, Z, C, MO, V, NEG, TPRME, B, DS, BMAX
      INTEGER I, J
      DIMENSION TPRME(200), B(200), DS(300), BMAX(300)
      DATA TPRME/200*0.0/, B/200*0.0/, DS/300*0.0/, BMAX/300*0.0/
C
      OPEN (UNIT=4,FILE='TERMINAL')
      OPEN (UNIT=7,FILE='BVSPLT DATA A')
C
C    ************************************************************************
C    * N IS THE INDEX OF REFRACTION. IAMP IS THE ACCELERATOR CURRENT        *
C    * IN AMPS. ENER IS THE ELECTRON ENERGY IN MEV. RISETM IS THE           *
C    * PULSE RISETIME IN SECONDS. S IS THE OBSERVER RADIAL DISTANCE IN      *
C    * METERS. Z IS THE OBSERVER AXIAL DISTANCE IN METERS. CO IS THE        *
C    * VACUUM SPEED OF LIGHT IN METERS PER SECONDS. MO IS THE ELECTRON      *
C    * MASS IN KILOGRAMS. ROE IS THE LINEAR CHARGE DENSITY IN COULOMBS      *
C    * PER METER.                                                           *
C    ************************************************************************
C
      N = 1.000293
      IAMP = 510.0
      ENER = 30.0
      RISETM = 10.0E-12
      S = 0.0
      Z = 0.0
C
      CO = 2.99792458E08
      MO = 9.109534E-31
      C = CO/N
      ENERJ = ENER*1.6021892E-13
      V = CO*SQRT(1.0-((MO**2.0)*(CO**4.0)/(ENERJ**2.0)))
      IF (V.LE.C) GO TO 80
      BETA = V/CO
      ROE = IAMP/V
      BPRME = N*BETA
C
```

```
C     **********************************************************************
C     * SETTING U1 ARBITRARILY TO ZERO FIXES U2. THIS IS TO SIMPLIFY      *
C     * CALCULATIONS.                                                     *
C     **********************************************************************
C
        U1 = 0.0
        U2 = -V*RISETM
C
C     **********************************************************************
C     * WITH 200 POINTS, TINC WAS SELECTED SO AS FOR THE PULSE MAXIMUM    *
C     * TO OCCUR AT APPROXIMATELY 20% OF THE GRAPH LENGHT.                *
C     **********************************************************************
C
        TINC = RISETM/40.0
        BPSQ = BPRME**2.0
        D = Z*BPSQ
        DD = 1 - BPSQ
C
        DO 60 J = 1,300
           S = S + 0.01
           DS(J) = S
           SSQ = S**2.0
           NEG = SSQ*DD
C
C     **********************************************************************
C     * ZPMIN IS THE Z VALUE OF THE MINIMUM OF THE U CURVE.               *
C     **********************************************************************
C
           ZPMIN = Z - (S/(SQRT(BPSQ - 1.0)))
C
C     **********************************************************************
C     * T1 AND T2 ARE THE TIMES THAT THE U CURVE MINIMUM IS TANGENT TO    *
C     * THE U1 AND U2 LEVELS RESPECTIVELY.                                *
C     **********************************************************************
C
           T1 = (ZPMIN - U1)/V + (SQRT(SSQ + (Z - ZPMIN)**2.0))/C
           T2 = T1 + RISETM
C
           DO 40 I = 1,200
              IF (I.EQ.1) THEN
                 TPRME(I) = T1
                 B(I) = 0.0
              ELSE
                 TPRME(I) = T1 + (REAL(I)*TINC)
              END IF
C
C     **********************************************************************
C     * THIS IF STATEMENT HANDLES THE CHANGEOVER IN LIMITS OF INTEGRAT-   *
C     * ION FROM ONE INTERVAL TO TWO INTERVALS.                           *
C     **********************************************************************
C
              IF (TPRME(I).GT.T2) GO TO 10
              A1 = U1 + V*TPRME(I)
              E1 = (Z - A1)**2.0
C
C     **********************************************************************
```

```
C    * THIS IF STATEMENT HANDLES THE CASE OF HAVING A NEGATIVE SQUARE  *
C    * ROOT IN REAL SPACE WHEN DETERMINING THE LIMITS OF INTEGRATION.  *
C    *******************************************************************
C
           IF (E1.LE.ABS(NEG)) THEN
              B(I) = 0.0
              GO TO 30
           ELSE
              Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
              Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
           ENDIF
C
           WPI = Z - Z1I
           WPF = Z - Z1F
           W1 = WPI/S
           W2 = WPF/S
           YY = ATAN(W1)
           XX = ATAN(W2)
           B(I) = -ROE*N*(BETA**2.0)*(YY-XX)*1.0E04/U2
           GO TO 20
C
   10      CONTINUE
           A1 = U1 + V*TPRME(I)
           A2 = U2 + V*TPRME(I)
           E1 = (Z - A1)**2.0
           E2 = (Z - A2)**2.0
           IF (E1.LE.ABS(NEG)) THEN
              B(I) = 0.0
              GO TO 30
           ELSE
              Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
              Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
           ENDIF
C
           IF (E2.LE.ABS(NEG)) THEN
              B(I) = 0.0
              GO TO 30
           ELSE
              Z2I = ((A2-D) + SQRT(BPSQ*(E2+NEG)))/DD
              Z2F = ((A2-D) - SQRT(BPSQ*(E2+NEG)))/DD
           ENDIF
C
           WPI = Z - Z1I
           WPF = Z - Z2I
           W1 = WPI/S
           W2 = WPF/S
           YY = ATAN(W1)
           XX = ATAN(W2)
           B1 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
C
           WPI = Z - Z2F
           WPF = Z - Z1F
           W1 = WPI/S
           W2 = WPF/S
           YY = ATAN(W1)
           XX = ATAN(W2)
```

```
            B2 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
            B(I) = B1 + B2
C
   20       CONTINUE
C
   30       CONTINUE
C
   40    CONTINUE
C
         YMAX = -1.0E20
         DO 50 I = 1,200
            IF (B(I).GT.YMAX) YMAX = B(I)
   50    CONTINUE
         BMAX(J) = YMAX
   60 CONTINUE
C
      DO 70 I = 1,300
         WRITE(7,1000) DS(I),BMAX(I)
 1000    FORMAT(G15.7,G15.7)
   70 CONTINUE
C
      GO TO 90
   80 WRITE(4,1010)
 1010 FORMAT(/,'V IS LESS THAN C, THEREFORE THERE IS NO CERENKOV RADIATI
     $ON FOR THIS ELECTRON',4X,' ENERGY')
   90 CONTINUE
C
      STOP
      END
C
C
C
      PROGRAM BVSPLT
C
      REAL X(300),Y(300),XMIN,XMAX,YMIN,YMAX
C     DOUBLE PRECISION X(300),Y(300),XMIN,XMAX,YMIN,YMAX
      INTEGER N
      CHARACTER*25 TITLE$
C
      TITLE$ = 'B MAX VS RADIAL DISTANCE$'
      N = 300
C
      READ(7,8000) ((X(I),Y(I)),I=1,N)
 8000 FORMAT(2G15.7)
C
C     CALL TEK618
      CALL SHERPA('P9      ','B',3)
      CALL PAGE(6,7.5)
      CALL AREA2D(4.5,5.5)
      CALL XNAME('RADIAL DISTANCE (METERS)$',100)
      CALL YNAME('CERENKOV PULSE (GAUSS)$',100)
      CALL HEADIN(TITLE$,100,1.5,1)
      CALL CROSS
      CALL RANGE(X,XMIN,XMAX,N)
      CALL RANGE1(Y,YMIN,YMAX,N)
      CALL GRAF(XMIN,'SCALE',XMAX,YMIN,'SCALE',YMAX)
```

```
      CALL CURVE(X,Y,300,0)
      CALL ENDPL(0)
      CALL DONEPL
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
      YMAX=-1.E20
      DO 10 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   10 CONTINUE
C
      RETURN
      END
C
C
C
      SUBROUTINE RANGE1(Y,YMIN,YMAX,N)
C
      DIMENSION Y(N)
      YMIN=1.E20
      YMAX=-1.E20
      DO 20 I=1,N
         IF(Y(I).GT.YMAX) YMAX= Y(I)
         IF(Y(I).LT.YMIN) YMIN= Y(I)
   20 CONTINUE
C
      RETURN
      END
```

APPENDIX D. S - T - B PROGRAM

```
      PROGRAM STB
C
C    ***********************************************************************
C    * THIS PROGRAM IS LOCATED ON THE NPS MAIN FRAME COMPUTER AND IS      *
C    * WRITTEN USING WATFOR 77. AFTER ENTERING CERTAIN PARAMETERS, THE    *
C    * SHAPE OF THE CERENKOV PULSE IS GENERATED. THE S VALUE IS INCRE-    *
C    * MENTED AND THE PROCESS IS REPEATED. THESE VALUES ARE STORED FOR    *
C    * LATER USE IN A 3-D PLOTTING ROUTINE.                               *
C    ***********************************************************************
C
      REAL N, U1, U2, BETA, CO, ROE, A1, A2, BPRME
      REAL RISETM, D, DD, E1, E2, Z1I, Z1F, Z2I, Z2F
      REAL B1, B2, W1, W2, YY, XX, T1, T2, WPI, WPF
      REAL ZPMIN, SSQ, BPSQ, TINC, IAMP, ENER, ENERJ
      REAL S, Z, C, MO, V, NEG, TPRME, B
      REAL BF, SRD, TIME
      INTEGER I, J, K, L, M
      DIMENSION TPRME(200), B(200)
      DIMENSION BF(2500), SRD(2500), TIME(2500)
      DATA TPRME/200*0.0/, B/200*0.0/, M/2500/
      DATA BF/2500*0.0/, SRD/2500*0.0/, TIME/2500*0.0/
C
      OPEN (UNIT=4,FILE='TERMINAL')
      OPEN (UNIT=7,FILE='S DATA B')
      OPEN (UNIT=8,FILE='T DATA B')
      OPEN (UNIT=9,FILE='B DATA B')
C
C    ***********************************************************************
C    * N IS THE INDEX OF REFRACTION. IAMP IS THE ACCELERATOR CURRENT      *
C    * IN AMPS. ENER IS THE ELECTRON ENERGY IN MEV. RISETM IS THE         *
C    * PULSE RISETIME IN SECONDS. S IS THE OBSERVER RADIAL DISTANCE IN    *
C    * METERS. Z IS THE OBSERVER AXIAL DISTANCE IN METERS. CO IS THE      *
C    * VACUUM SPEED OF LIGHT IN METERS PER SECONDS. MO IS THE ELECTRON    *
C    * MASS IN KILOGRAMS. ROE IS THE LINEAR CHARGE DENSITY IN COULOMBS    *
C    * PER METER.                                                         *
C    ***********************************************************************
C
      N = 1.000293
      IAMP = 510.0
      ENER = 30.0
      RISETM = 10.0E-12
      S = 0.0
      Z = 0.0
C
      CO = 2.99792458E08
      MO = 9.109534E-31
      C = CO/N
      ENERJ = ENER*1.6021892E-13
      V = CO*SQRT(1.0-((MO**2.0)*(CO**4.0)/(ENERJ**2.0)))
      IF (V.LE.C) GO TO 90
      BETA = V/CO
```

```
      ROE = IAMP/V
      BPRME = N*BETA
C
C    ************************************************************************
C    * SETTING U1 ARBITRARILY TO ZERO FIXES U2. THIS IS TO SIMPLIFY        *
C    * CALCULATIONS.                                                       *
C    ************************************************************************
C
      U1 = 0.0
      U2 = -V*RISETM
C
C    ************************************************************************
C    * WITH 200 POINTS, TINC WAS SELECTED SO AS FOR THE PULSE MAXIMUM     *
C    * TO OCCUR AT APPROXIMATELY 20% OF THE GRAPH LENGHT.                  *
C    ************************************************************************
C
      TINC = RISETM/10.0
      BPSQ = BPRME**2.0
      D = Z*BPSQ
      DD = 1 - BPSQ
C
      DO 50 K = 1,50
         IF (K.EQ.1) THEN
            J = 0
         ELSE
            J = 50*(K-1)
         END IF
         S = S + 0.5
         SSQ = S**2.0
         NEG = SSQ*DD
C
C    ************************************************************************
C    * ZPMIN IS THE Z VALUE OF THE MINIMUM OF THE U CURVE.                 *
C    ************************************************************************
C
         ZPMIN = Z - (S/(SQRT(BPSQ - 1.0)))
C
C    ************************************************************************
C    * T1 AND T2 ARE THE TIMES THAT THE U CURVE MINIMUM IS TANGENT TO     *
C    * THE U1 AND U2 LEVELS RESPECTIVELY.                                  *
C    ************************************************************************
C
         T1 = (ZPMIN - U1)/V + (SQRT(SSQ + (Z - ZPMIN)**2.0))/C
         T2 = T1 + RISETM
C
         DO 40 I = 1,50
            IF (I.EQ.1) THEN
               TPRME(I) = T1
               B(I) = 0.0
            ELSE
               TPRME(I) = T1 + (REAL(I)*TINC)
            END IF
            L = J + I
            SRD(L) = S
            TIME(L) = TPRME(I)
C
```

```
C     **********************************************************************
C     * THIS IF STATEMENT HANDLES THE CHANGEOVER IN LIMITS OF INTEGRAT- *
C     * ION FROM ONE INTERVAL TO TWO INTERVALS.                          *
C     **********************************************************************
C
            IF (TPRME(I).GT.T2) GO TO 10
            A1 = U1 + V*TPRME(I)
            E1 = (Z - A1)**2.0
C
C     **********************************************************************
C     * THIS IF STATEMENT HANDLES THE CASE OF HAVING A NEGATIVE SQUARE  *
C     * ROOT IN REAL SPACE WHEN DETERMINING THE LIMITS OF INTEGRATION.  *
C     **********************************************************************
C
            IF (E1.LE.ABS(NEG)) THEN
               B(I) = 0.0
               GO TO 30
            ELSE
               Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
               Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
            ENDIF
C
            WPI = Z - Z1I
            WPF = Z - Z1F
            W1 = WPI/S
            W2 = WPF/S
            YY = ATAN(W1)
            XX = ATAN(W2)
            B(I) = -ROE*N*(BETA**2.0)*(YY-XX)*1.0E04/U2
            GO TO 20
C
   10       CONTINUE
            A1 = U1 + V*TPRME(I)
            A2 = U2 + V*TPRME(I)
            E1 = (Z - A1)**2.0
            E2 = (Z - A2)**2.0
            IF (E1.LE.ABS(NEG)) THEN
               B(I) = 0.0
               GO TO 30
            ELSE
               Z1I = ((A1-D) + SQRT(BPSQ*(E1+NEG)))/DD
               Z1F = ((A1-D) - SQRT(BPSQ*(E1+NEG)))/DD
            ENDIF
C
            IF (E2.LE.ABS(NEG)) THEN
               B(I) = 0.0
               GO TO 30
            ELSE
               Z2I = ((A2-D) + SQRT(BPSQ*(E2+NEG)))/DD
               Z2F = ((A2-D) - SQRT(BPSQ*(E2+NEG)))/DD
            ENDIF
C
            WPI = Z - Z1I
            WPF = Z - Z2I
            W1 = WPI/S
            W2 = WPF/S
```

```
            YY = ATAN(W1)
            XX = ATAN(W2)
            B1 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
C
            WPI = Z - Z2F
            WPF = Z - Z1F
            W1 = WPI/S
            W2 = WPF/S
            YY = ATAN(W1)
            XX = ATAN(W2)
            B2 = -ROE*N*(BETA**2.0)*(YY - XX)*1.0E04/U2
            B(I) = B1 + B2
C
   20       CONTINUE
C
   30       CONTINUE
C
            BF(L) = B(I)
   40    CONTINUE
C
   50 CONTINUE
C
      DO 60 I = 1,M
         WRITE(7,1000) SRD(I)
 1000    FORMAT(G15.7)
   60 CONTINUE
C
      DO 70 I = 1,M
         WRITE(8,1000) TIME(I)
   70 CONTINUE
C
      DO 80 I = 1,M
         WRITE(9,1000) BF(I)
   80 CONTINUE
C
      GO TO 100
   90 WRITE(4,1010)
 1010 FORMAT(/,'V IS LESS THAN C, THEREFORE THERE IS NO CERENKOV RADIATI
     $ON FOR THIS ELECTRON',4X,' ENERGY')
  100 CONTINUE
C
      STOP
      END
C
C
C
      PROGRAM THREED
C
      REAL S(2500),T(2500),B(2500),X3MAX,Y3MAX,Z3MAX,ZMAT(2500)
      INTEGER I,N
      CHARACTER*35 TITLE$
C
      TITLE$ = 'B-FIELD VS TIME VS RADIAL DISTANCE$'
      N = 2500
C
      READ(7,8000) (S(I),I=1,N)
```

```
 8000 FORMAT(G15.7)
      READ(8,8000) (T(I),I=1,N)
      READ(9,8000) (B(I),I=1,N)
C
      CALL RANGE(S,X3MAX,N)
      CALL RANGE(T,Y3MAX,N)
      CALL RANGE(B,Z3MAX,N)
C
C     CALL TEK618
      CALL SHERPA('P10      ','B',3)
      CALL PAGE(7.5,9.0)
      CALL AREA2D(5.5,7.0)
      CALL VOLM3D(1,1,1)
      CALL X3NAME('DISTANCE (METERS)$',100)
      CALL Y3NAME('TIME (SECONDS)$',100)
      CALL Z3NAME('MAGNITUDE (GAUSS)$',100)
      CALL HEADIN(TITLE$,100,1.0,1)
      CALL VUABS(1.0,-0.5,0.5)
      CALL GRAF3D(0.0,'SCALE',X3MAX,0.0,'SCALE',Y3MAX,0.0,'SCALE',Z3MAX)
      CALL BGNMAT(50,50)
      CALL GETMAT(S,T,B,2500,0)
      CALL ENDMAT(ZMAT,0)
      CALL SURMAT(ZMAT,1,50,5,50,0)
      CALL ENDPL(0)
      CALL DONEPL
C
      STOP
      END
C
C
C
      SUBROUTINE RANGE(Y,YMAX,N)
C
      DIMENSION Y(N)
      YMAX = -1.0E20
      DO 20 I = 1,N
         IF (Y(I).GT.YMAX) YMAX = Y(I)
   20 CONTINUE
C
      RETURN
      END
```

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